

Name Key 2017

Math 3 – Sequences and Series Review

Determine whether the sequence is arithmetic, geometric, or neither. If arithmetic, find the common difference. If geometric, find the common ratio. If neither, explain why.

1. $5, 7, 9, 11, 13, \dots$

Arithmetic
 $d=2$

2. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots$

Neither!

no common
difference
or common
ratio

3. $\frac{16}{9}, \frac{8}{3}, 4, 6, 9, \dots$

Geometric
 $r = 3/2$

Identify if the formula is explicit or recursive, then use the formula to generate the first 5 terms of the sequence.

Then write the formula in the form not provided (so if it's explicit, write the recursive form and vice versa).

4. $a_n = \frac{1}{4}n - 12$ **Explicit**

5. $a_n = a_{n-1} \cdot 4, a_1 = -3$ **Recursive**

6. $a_n = a_{n-1} + 6, a_1 = 55$ **Recursive**

$-11.75, -11.5, -11.25, -11, -10.75, -3, -12, -48, -192, -768, 55, 61, 67, 73, 79$

Recursive:
 $a_1 = -11.75$

$a_n = a_{n-1} + \frac{1}{4}$

Explicit:

$a_n = -3(4)^{n-1}$

Explicit: $a_n = 55 + (n-1)(6)$

*must distribute & combine like terms
 $a_n = 55 + 6n - 6$

$a_n = 6n + 49$

Write the explicit and recursive formula for each sequence.

7. $3, -7, -17, -27, \dots$ **arithmetic** $d = -10$

$a_n = 3 + (n-1)(-10)$

$a_n = 3 - 10n + 10$

$a_n = -10n + 13$

Explicit: $a_n = -10n + 13$

Recursive: $a_1 = 3$ $a_n = a_{n-1} - 10$

8. $-4, 12, -36, 108, \dots$ **geometric** $r = -3$

Explicit: $a_n = -4(-3)^{n-1}$

Recursive: $a_1 = -4$ $a_n = a_{n-1} \cdot -3$

Find the missing term in the following **arithmetic** sequences. (Hint – solve for “ d ”)

$a_n = a_1 + (n-1)d$

9. $10, a_2, 20$
 $a_1 = 10$ $a_3 = 20$
 $20 = 10 + (3-1)d$
 $10 = 2d$
 $d = 5 \rightarrow a_2 = 15$

10. $-2.1, \frac{-2.3}{a_1}, \frac{-2.5}{a_2}, -2.7$
 $-2.7 = -2.1 + (4-1)(d)$
 $-0.6 = 3d$
 $-0.2 = d \rightarrow$ now add -0.2 to the first term!

11. $a_1 = -2, \frac{10}{a_2}, \frac{-10}{a_3}, 50$
 $-50 = -2(r)^{3-1}$
 $25 = r^2$
 $\pm 5 = r$

12. $a_1 = 100, \frac{30}{a_2}, \frac{9}{a_3}, 2.7$
 $2.7 = 100(r)^{4-1}$
 $0.027 = r^3$
 $0.3 = r$

Find the missing term in the following **geometric** sequences. (Hint – solve for “ r ”)

$a_n = a_1(r)^{n-1}$

11. $a_1 = -2, \frac{10}{a_2}, \frac{-10}{a_3}, 50$
 $-50 = -2(r)^{3-1}$
 $25 = r^2$
 $\pm 5 = r$

12. $a_1 = 100, \frac{30}{a_2}, \frac{9}{a_3}, 2.7$
 $2.7 = 100(r)^{4-1}$
 $0.027 = r^3$
 $0.3 = r$

Find the term stated in each problem. Use the appropriate formula, SHOW ALL WORK!

- Geometric! $r=2$
13. Find the 33rd term: 6, 12, 24, 48 ... $a_n = a_1(r)^{n-1}$

$$a_{33} = 6(2)^{33-1}$$

$$a_{33} = 2.577 \times 10^{10}$$

or

$$a_{33} = 25769803780$$

- Arithmetic! $d=2$
14. Find the 56th term: -21, -19, -17 ... $a_n = a_1 + (n-1)d$

$$a_{56} = -21 + (56-1)(2)$$

$$a_{56} = 89$$

15. How many terms are in the sequence: 15, 12, 9, ... -39
Arithmetic! $a_n = a_1 + (n-1)d$ a_1, a_2, a_3, a_n
 $d=-3$ * we know -39 is the last term, but we don't know which term number it is! solve for n!

$$-39 = 15 + (n-1)(-3)$$

$$-54 = (n-1)(-3)$$

$$\begin{aligned} 18 &= n-1 \\ 19 &= n \end{aligned}$$

so -39 is the 18th term

17. In a geometric sequence, the 3rd term is 5 and the common ratio is 4. Find the 7th term.
 $a_n = a_1(r)^{n-1}$

$$\begin{aligned} a_3 &= 5 \\ r &= 4 \\ a_7 &=? \end{aligned}$$

* since we don't know the first term, manipulate the equation to be about the first term we know!

$$a_7 = a_3(4)^{7-3}$$

$$a_7 = 5(4)^{7-3}$$

$$a_7 = 1280$$

the 7th term is 1280

19. Find the sum of the series: -12, -17, -23, ... -67

Arithmetic! $d = -5$

$$S_n = \frac{n}{2}(-12 + -67)$$

* we know the last term but we don't know its term #! Go back to sequence!

$$-55 = (n-1)(-5)$$

$$11 = n-1$$

12 = n → so -67 is the 12th term!

$$S_{12} = \frac{12}{2}(-12 + -67) = -474$$

16. Which term is $\frac{1}{64}$ in the sequence 256, 128, 64, ...
Geometric! $a_n = a_1(r)^{n-1}$ $r = \frac{1}{2}$

$$\frac{1}{64} = 256\left(\frac{1}{2}\right)^{n-1} \rightarrow 14 = n-1$$

$$\frac{1}{64} = \left(\frac{1}{2}\right)^{n-1}$$

$$\log_{\frac{1}{2}} \frac{1}{64} = n-1$$

18. In an arithmetic sequence, the 7th term is 11 and the common difference is -3. Find the 53rd term.

$$\begin{aligned} a_7 &= 11 & a_{53} &= a_1 + (53-1)(-3) \\ d &= -3 & a_{53} &=? \end{aligned}$$

* same as #17! manipulate the equation!

$$a_{53} = a_7 + (53-7)(-3)$$

$$a_{53} = 11 + (53-7)(-3)$$

$$a_{53} = -127$$

20. Find the sum of the first 17 terms of the series

$$18 + 25 + 32 + 39 + \dots$$

$$S_{17} = \frac{17}{2}(18 + a_{17})$$

* we don't know the last term! go back to sequence!

$$\text{Sequence: } a_{17} = 18 + (17-1)(7)$$

$$a_{17} = 130$$

$$\text{now find the sum: } S_{17} = \frac{17}{2}(18 + 130) = 1258$$

21. Find the sum of the first 12 terms of the sequence

$$r = -3 \quad 4, -12, 36, -108, \dots \quad \text{Geometric} \quad S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$S_{12} = 4 \left(\frac{1-(-3)^{12}}{1-(-3)} \right) \quad * \text{be very careful} \\ \text{plugging this} \\ \text{into the calculator!}$$

$$S_{12} = -531440$$

22. Find the sum of the first 9 terms of the sequence

$$4374, 1458, 486, 162, \dots \quad \text{Geometric} \quad S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$r = \sqrt[3]{3} \quad S_9 = 4374 \left(\frac{1-(\sqrt[3]{3})^9}{1-\sqrt[3]{3}} \right)$$

$$S_9 = 6560.667 \text{ or} \\ \frac{19682}{3}$$

23. How many terms must be added to give a sum of -4665 ? 24. How many terms must be added so that $s_n = -220$?

$$-3, -18, -108, \dots \quad \text{Geometric} \quad S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$$

$$-4665 = -3 \left(\frac{1-6^n}{1-6} \right)$$

$$1555 = \frac{1-6^n}{-5}$$

$$-7775 = 1-6^n$$

$$-7776 = -6^n$$

$$7776 = 6^n$$

$$\log_6 7776 = n$$

$$n=5$$

$$-220 = \frac{n}{2} (-5 + a_n) \quad \text{Arithmetic} \quad S_n = \frac{n}{2}(a_1 + a_n)$$

go back to sequence formula

$$\text{sequence: } a_n = -5 + (n-1)(-3)$$

$$a_n = -5 - 3n + 3$$

$$a_n = -3n - 2 \quad * \text{now plug this} \quad \text{into series formula}$$

series:

$$-220 = \frac{n}{2} (-5 - 3n - 2)$$

$$-440 = n(-3n - 7)$$

$$-440 = -3n^2 - 7n$$

$$0 = -3n^2 - 7n + 440$$

*graph in calc & look in table for $y=0$

$$n=11$$

For each series find "r" and state whether it is convergent or divergent and find the infinite sum (if it exists). *If there is no infinite sum, write "NO SUM."

$$25. 36 + 12 + 4 + \dots$$

$$r = \frac{1}{3}$$

convergent

$$S = \frac{36}{1-\frac{1}{3}}$$

$$S = 54$$

$$26. 250 - 50 + 10 - \dots$$

$$r = -\frac{1}{5}$$

convergent

$$S = \frac{250}{1-(-\frac{1}{5})}$$

$$S = \frac{625}{3}$$

$$27. \frac{1}{48} - \frac{1}{24} + \frac{1}{12} - \dots$$

$$r = -2$$

divergent

no infinite sum

Find the specified value for the infinite geometric series that is described.

$$28. r = \frac{1}{3}, \quad S = 15, \quad a_1 = \underline{10}$$

$$15 = \frac{a_1}{1-\frac{1}{3}}$$

$$15 = \frac{a_1}{\frac{2}{3}}$$

$$a_1 = 10$$

$$29. r = -\frac{1}{4}, \quad S = 24, \quad a_1 = \underline{30}$$

$$24 = \frac{a_1}{1-(-\frac{1}{4})}$$

$$24 = \frac{a_1}{\frac{5}{4}}$$

$$a_1 = 30$$

Solve each problem using arithmetic or geometric sequences.

$$a_1 = 17$$

$$d = .30$$

$$\begin{cases} S_n = \frac{n}{2}(a_1 + a_n) \\ a_n = a_1 + (n-1)d \end{cases}$$

30. Max has a change jar with \$17. Every school day he plans to add the 30¢ change from his lunch money to the jar. How much money will Max have at the end of the school year (180 days)? **Arithmetic**

total amount \rightarrow find the sum!

$$S_{180} = \frac{180}{2}(17 + a_{180})$$

* go back to sequence formula to find a_{180}

$$\text{sequence: } a_{180} = 17 + (180-1)(.30)$$

$$S_{180} = \frac{180}{2}(17 + 70.70)$$

$$a_{180} = 70.70 * \text{now plug this back into series formula!}$$

$$S_{180} = \$7,893$$

31. Antonio decided to start saving his money. If he saves \$0.25 the first week, \$0.50 for the second week, \$1 for the third week, \$2 for the fourth week, and so on, how much would he have saved after 20 weeks?

$a_1 = .25$ { geometric $\rightarrow S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$ total amount \rightarrow find the sum!

$$a_2 = .50$$

$$r = 2$$

$$a_3 = 1$$

$$a_4 = 2$$

$$S_{20} = .25 \left(\frac{1-2^{20}}{1-2} \right)$$

$$S_{20} = \$262143.75$$

32. A layered sculpture is arranged so that there are 5 diamonds on the top design layer, 7 diamonds on the second layer, 9 diamonds on the third layer, and so on. How many diamonds are on the twentieth layer?

$$a_{20} = ?$$

$a_1 = 5$ { arithmetic $\rightarrow a_n = a_1 + (n-1)d$

$$a_2 = 7$$

$$a_3 = 9$$

$$a_{20} = 5 + (20-1)(2)$$

$$a_{20} = 43 \text{ diamonds}$$

33. In a school auditorium, there are 20 rows of seats. There are 22 seats in the first row, 24 in second row, 26 in the third row, and so on. How many seats are in the 20th row? How many total seats are in the auditorium?

$$n = 20$$

$$a_1 = 22$$

$$a_2 = 24$$

$$a_3 = 26$$

Arithmetic

$$a_{20} = ?$$

$$a_n = a_1 + (n-1)d$$

$$a_{20} = 22 + (20-1)(2)$$

$$a_{20} = 60$$

60 seats in the 20th row

$$S_{20} = ?$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{20} = \frac{20}{2}(22+60)$$

$$S_{20} = 820$$

820 seats in the theater