M3 HW 3 – ARITHMETIC SERIES NAME key Spring 17

1. In an arithmetic series, $a_1 = 7$ and $a_{12} = 29$.

   Find s_{12}.

   $S_{12} = \frac{12}{2}(7+29)$ $\boxed{S_{12} = 216}$

2. In an arithmetic series, $a_1 = -12$ and $a_{14} = 54$.

   Find s_{14}.

   $S_{14} = \frac{14}{2}(-12+54)$ $\boxed{S_{14} = 294}$

3. What is the sum of the series:

   $3 + 5 + 7 + 9 + \cdots + 57$

   $a_1 = 3$
   $d = 2$
   $a_n = 57$

   $s_n = \frac{n}{2}(3+57)$

   sequence: $57 = 3 + (n-1)(2)$
   $54 = (n-1)(2)$
   $27 = n-1$
   $28 = n$

   series: $S_{28} = \frac{28}{2}(3+57)$ $\boxed{S_{28} = 840}$

4. Find the sum of the series:

   $1 - 3 - 7 - 11 \ldots - 51$

   $a_1 = 1$
   $d = -4$
   $a_n = -51$

   $S_n = \frac{n}{2}(1-51)$

   sequence: $-51 = 1 + (n-1)(-4)$
   $-52 = (n-1)(-4)$
   $13 = n-1$
   $14 = n$

   series: $S_{14} = \frac{14}{2}(1-51)$ $\boxed{S_{14} = -350}$

5. In an arithmetic series, find the sum of the first 72 terms

   If the first term is 5 and the common difference is $\frac{1}{3}$.

   $n = 72$
   $a_1 = 5$
   $d = 1/3$

   $S_{72} = \frac{72}{2}(5 + a_{72})$

   sequence: $a_{72} = 5 + (72-1)(1/3)$
   $a_{72} = 28.\overline{66}$ or $86/3$

   series: $S_{72} = \frac{72}{2}\left(5 + \frac{86}{3}\right)$ $\boxed{S_{72} = 1212}$

6. In an arithmetic series, find the sum of the first 10

   Terms if the first term is 3 and d = 4.

   $n = 10$
   $a_1 = 3$
   $d = 4$

   $S_{10} = \frac{10}{2}(3 + a_{10})$

   sequence: $a_{10} = 3 + (10-1)(4)$
   $a_{10} = 39$

   series: $S_{10} = \frac{10}{2}(3+39)$ $\boxed{S_{10} = 210}$

7. If $a_6 = -5$ and $a_{10} = 7$ in an arithmetic series, find the

   Sum of the first 12 terms. $S_{12} = \frac{12}{2}(a_1 + a_{12})$

   manipulate formula:
   $a_n = a_6 + (n-6)d$
   $a_n = -5 + (n-6)d$

   Find d using other term:
   $7 = -5 + (10-6)d$
   $12 = 4d$
   $3 = d$

   update eqn:
   $a_n = -5 + (n-6)(3)$

   (4) find a_1:
   $a_1 = -5 + (1-6)(3)$
   $a_1 = -20$

   (5) find a_{12}:
   $a_{12} = -5 + (12-6)(3)$
   $a_{12} = 13$

   (6) find S_{12}:
   $S_{12} = \frac{12}{2}(-20+13)$
   $\boxed{S_{12} = -42}$

8. If $a_5 = 16$ and $a_{11} = 4$ in an arithmetic series, find

   The sum of the first 20 terms. $S_{20} = \frac{20}{2}(a_1 + a_{20})$

   (1) manipulate formula: $a_n = 16 + (n-5)d$

   (2) find d using other term:
   $4 = 16 + (11-5)d$
   $-12 = 6d$
   $-2 = d$

   (3) update eqn:
   $a_n = 16 + (n-5)(-2)$

   (4) find a_1:
   $a_1 = 16 + (1-5)(-2)$
   $a_1 = 24$

   (5) find a_{20}:
   $a_{20} = 16 + (20-5)(-2)$
   $a_{20} = -14$

   (6) find S_{20}:
   $S_{20} = \frac{20}{2}(24-14)$
   $\boxed{S_{20} = 100}$

$a_1 = 10 \quad d = 1 \quad a_n = 53$

9. Find the sum of the integers 10 through 53.

$S_n = \frac{n}{2}(10 + 53)$

sequence: $53 = 10 + (n-1)(1)$

$43 = (n-1)(1)$

$43 = n - 1$

$44 = n$

series: $S_{44} = \frac{44}{2}(10 + 53)$

$\boxed{S_{44} = 1386}$

$a_1 = -14 \quad d = 2 \quad a_n = 22$

10. Find the sum of the even integers −14 through 22.

$S_n = \frac{n}{2}(-14 + 22)$

sequence: $22 = -14 + (n-1)(2)$

$36 = (n-1)(2)$

$18 = n - 1$

$19 = n$

series: $S_{19} = \frac{19}{2}(-14 + 22)$

$\boxed{S_{19} = 76}$

11. How many terms of the sequence must be added to

Get a sum of 801? $a_1 = -15$

$-15 + -8 + -1 + \ldots \quad d = +7$

$S_n = 801$

$801 = \frac{n}{2}(-15 + a_n)$

sequence: $a_n = -15 + (n-1)(7)$

$a_n = -15 + 7n - 7$

$a_n = 7n - 22$

$801 = \frac{n}{2}(-15 + 7n - 22)$

$1602 = n(7n - 37)$

$1602 = 7n^2 - 37n$

$0 = 7n^2 - 37n - 1602$

☆Graph in calc☆

$\boxed{n = 18}$

12. How many terms of the series must be added to

Get a sum of −2070?

18 + 12 + 6 + ⋯

$a_1 = 18$

$d = -6$

$S_n = -2070$

$-2070 = \frac{n}{2}(18 + a_n)$

sequence: $a_n = 18 + (n-1)(-6)$

$a_n = 18 - 6n + 6$

$a_n = -6n + 24$

$-2070 = \frac{n}{2}(18 - 6n + 24)$

$-4140 = n(-6n + 42)$

$-4140 = -6n^2 + 42n$

$6n^2 - 42n - 4140 = 0$

★Graph in calc☆

$\boxed{n = 30}$