GEOMETRIC SERIES HOMEWORK

NAME key 2017

Find the sum of each series.

1. $-1, -6, -36, -216$

$a_1 = -1 \quad r = 6 \quad n = 4$

$S_4 = -1\left(\frac{1-6^4}{1-6}\right)$

$\boxed{S_4 = -259}$

2. $2, 6, 18, 54, 162$

$a_1 = 2 \quad r = 3 \quad n = 5$

$S_5 = 2\left(\frac{1-3^5}{1-3}\right)$

$\boxed{S_5 = 242}$

3. $a_1 = -2,\ r = 4,\ n = 7$

$S_7 = -2\left(\frac{1-4^7}{1-4}\right)$

$\boxed{S_7 = -10{,}922}$

4. $-3 - 12 - 48 \ldots n = 6$ $\quad r = 4$

$S_6 = -3\left(\frac{1-4^n}{1-4}\right)$

$\boxed{S_6 = -4095}$

5. $-2 + 6 - 18 + 54 \ldots n = 8$ $\quad r = -3$

$S_8 = -2\left(\frac{1-(-3)^8}{1-(-3)}\right)$

$\boxed{S_8 = 3280}$

6. $a_1 = 128,\ r = \frac{1}{2},\ n = 4$

$S_4 = 128\left(\frac{1-(\frac{1}{2})^4}{1-(\frac{1}{2})}\right)$

$\boxed{S_4 = 240}$

7. $4 + 2 + 1 + \cdots \frac{1}{16}$ $\quad a_1 = 4 \quad r = 1/2 \quad a_n = 1/16$

$S_n = 4\left(\frac{1-(\frac{1}{2})^n}{1-(1/2)}\right)$

$\rightarrow S_7 = 4\left(\frac{1-(\frac{1}{2})^7}{1-(1/2)}\right)$

☆sequence formula:

$\frac{1}{16} = 4\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{64} = \left(\frac{1}{2}\right)^{n-1}$

$\log_{\frac{1}{2}} \frac{1}{64} = n-1$

$6 = n-1 \rightarrow n = 7$

$\boxed{S_7 = 7.9375 \text{ or } \frac{127}{16}}$

8. $\frac{1}{9} + \frac{1}{3} + 1 + \cdots 2187$

$S_n = \frac{1}{9}\left(\frac{1-3^n}{1-3}\right)$

$\rightarrow S_{10} = \frac{1}{9}\left(\frac{1-3^{10}}{1-3}\right)$

☆sequence formula:

$2187 = \frac{1}{9}(3)^{n-1}$

$19683 = 3^{n-1}$

$\log_3 19683 = n-1$

$9 = n-1$

$10 = n$

$\boxed{S_{10} = 3280.4 \text{ or } \frac{29524}{9}}$

Determine the number of terms in each series.

9. $a_1 = 3,\ r = -2,\ s_n = -15$

$-15 = 3\left(\frac{1-(-2)^n}{1-(-2)}\right)$

$-5 = \frac{1-(-2)^n}{3}$

$-15 = 1-(-2)^n$

$-16 = -(-2)^n$

$16 = (-2)^n$

$\log_2 16 = n$

$\boxed{n = 4}$

10. $a_1 = -3,\ r = 4,\ s_n = -63$

$-63 = -3\left(\frac{1-4^n}{1-4}\right)$

$21 = \frac{1-4^n}{-3}$

$-63 = 1-4^n$

$-64 = -4^n$

$64 = 4^n$

$\log_4 64 = n$

$\boxed{n = 3}$

11. $1+2+4+8\ldots s_n = 127$

$127 = 1\left(\frac{1-2^n}{1-2}\right)$

$127 = \frac{1-2^n}{-1}$

$-127 = 1-2^n$

$-128 = -2^n$

$128 = 2^n$

$\log_2 128 = n$

$\boxed{n=7}$

12. $2+8+32+128\ldots s_n = 2730$

$2730 = 2\left(\frac{1-4^n}{1-4}\right)$

$1365 = \frac{1-4^n}{-3}$

$-4095 = 1-4^n$

$-4096 = -4^n$

$4096 = 4^n$

$\log_4 4096 = n$

$\boxed{n=6}$

State whether each series will converge or diverges, then state the infinite sum if it exists.

13. $a_1 = 6.7 \;,\; r = 1.1$

diverge
no sum

14. $a_1 = \frac{5}{6} \;,\; r = \frac{3}{5}$

converge
$S = \frac{25}{12}$

$S = \frac{5/6}{1-3/5}$

15. $1 + \frac{1}{2} + \frac{1}{4} + \cdots$

converge
$S = 2$

$S = \frac{1}{1-1/2}$

16. $3+9+27+81+\cdots$

diverge
no sum

17. $1 - \frac{3}{5} + \frac{9}{25} - \frac{27}{125} + \cdots$

converge
$S = \frac{5}{8}$

$S = \frac{1}{1-(-3/5)}$

18. $32+16+8+\cdots$

converge
$S = 64$

$S = \frac{32}{1-1/2}$

Determine the common ratio of the geometric series given the infinite sum.

19. $a_1 = 3 \;,\; S = 2$

$2 = \frac{3}{1-r}$

$2(1-r) = 3$

$1-r = 3/2$

$-r = -1/2$

$\boxed{r = \frac{1}{2}}$

20. $a_1 = 1 \;,\; S = 1.25$

$1.25 = \frac{1}{1-r}$

$1.25(1-r) = 1$

$1-r = 4/5$

$-r = \frac{-1}{5}$

$\boxed{r = \frac{1}{5}}$