

Key

Math III Exponential/Geometric Series

Exponential functions are of the form $y = a(b)^x$, where a is the y-intercept or initial amount, and b is the growth/decay factor. If b is $b > 1$ it represents exponential growth and if $0 < b < 1$ it represents exponential decay.

Ex/ $f(x) = 5,236(1.08)^x$ exponential growth, growth rate is 8%

Ex/ $f(x) = 2,873(0.91)^x$ exponential decay, decay rate is 9%

Compounded interest uses the formula $A = p(1 + \frac{r}{n})^{nt}$, where p is the principle, r is the rate, t is the time, and n is the number of times the interest is compounded. (monthly $n = 12$, weekly $n = 52$, etc.)

Continuously compounded interest uses the formula $A = Pe^{rt}$, where p is the principle, r is the rate, and t is the time.

Mortgage Formula - monthly payment = $\frac{pi}{1-(1+i)^{-n}}$ where p is the principle, n is the number of total payments, i is the monthly interest rate.

Sum of finite geometric series is found by $S_n = \frac{a_1(1-r^n)}{1-r}$, where a_1 is the first term, r is the ratio (what each term is multiplied by to get to the next), n is term number the series stops.

Examples:

1. 288, -96, 32, ... What is the approximate value of the sum of the 7th term?

$$\frac{288(1 - (-\frac{1}{3})^7)}{1 - (-\frac{1}{3})} = 216.1$$

2. 360 + 480 + 640 + ... What is the approximate value of the sum of the 15th term?

$$\frac{360(1 - (\frac{4}{3})^{15})}{1 - \frac{4}{3}} = 79,737.39$$

3. What is the approximate value of the sum:

$$8 - \frac{8}{7} + \frac{8}{49} - \dots 8 \cdot \left(\frac{-1}{7}\right)^{2500} ? \quad \frac{8(1 - (-\frac{1}{7})^{2500})}{1 - (-\frac{1}{7})} = 7$$

4. Find the monthly payment of \$175,000 home on a 30 year mortgage with a 3.5% interest rate.

$$\begin{aligned} i &= 1.035^{1/12} = 1.00287 \\ i &= .00287 \\ n &= 360 \end{aligned}$$

$$\frac{175,000(.00287)}{1 - (1 + .00287)^{-360}} = \$780.37$$

5. Angela deposited \$3000 into a savings account earning 4% interest compounded continuously, how much will she have after 6 years?

$$\begin{aligned} &3000e^{.04(6)} \quad Pe^{rt} \\ &= 3813.75 \end{aligned}$$

6. Sam deposited \$5,500 into a savings account earning 5.6% interest compounded monthly. How many years had he been saving when the savings account has a balance of \$8599.52?

$$\begin{aligned} 8,599.52 &= 5,500 \left(1 + \frac{.056}{12}\right)^{12x} \\ \frac{8,599.52}{5,500} &= \frac{5,500}{5,500} (1.0046)^{12x} \\ 1.56 &= (1.0046)^{12x} \end{aligned}$$

$$\begin{aligned} \log_{1.0046} 1.56 &= 12x \\ 96.89 &= 12x \quad x = 8.07 \end{aligned}$$

7. Mary wants a dress that costs \$450 for the prom. So far she has saved \$275 and put it in a savings account for 1.5 years, what interest rate must she earn to have \$450 by prom? (compounded continuously)

$$\begin{aligned} \frac{450}{275} &= \frac{275e^{1.5x}}{275} \\ \ln 1.64 &= \frac{1.5x}{1.5} \\ x &= .329 = 32.9\% \end{aligned}$$

8. A board is made up of 9 squares. A certain number of pennies is placed in each square, following a geometric sequence. The first square has 1 penny, the second has 2 pennies, the third has 4 pennies, etc. When every square is filled, how many pennies will be used in total?

A. 521 (B.) 511 C. 256 D. 81

1	2	4
8	16	32
64	128	256