Key

Graphing Rational Functions

A **rational function** are functions where $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are both polynomials and $b(x) \neq 0$.

$f(x)$ has a **vertical asymptote** whenever $b(x) = 0$

$f(x)$ has at most one **horizontal asymptote**

- If the degree of the numerator is greater than the denominator, there is no horizontal asymptote.
- If the degree of the numerator is less than the denominator, the horizontal asymptote is the x-axis (y = 0)
- If the degree of the numerator equals the denominator, the horizontal asymptote is the line $y = \frac{leading\ coefficint\ of\ numerator}{leading\ coefficient\ of\ denominator}$

Points of discontinuity (hole) – Occurs when the numerator and denominator have a common factor.

Examples:

1. Describe the asymptotes and points of discontinuity of $f(x) = \frac{x^2+4x-5}{x+5} = \frac{(x+5)(x-1)}{x+5}$ hole

point of discontinuity (hole) $x=-5$

no horizontal asymptote

no vertical asymptote since the $(x+5)$'s cancel the result is $f(x)=x-1$ which is a line.

2. Given the function: $g(x) = \frac{(x-2)(3x+2)}{(x+4)(x-2)(x-6)}$

- What are the equations of the asymptotes of this function? numerator x^2 → denominator x^3 so horizontal asymptote $y=0$ (x-axis) vertical asymptotes $x=-4, x=6$
- Determine if there are any points of discontinuity. Explain why or why not. yes @ $x=2$ since the function has an $x-2$ in the numerator and denominator
- Describe the end behavior as x approaches - ∞ and as x approaches + ∞

$x \to -\infty \quad g(x) \to -\infty$

$x \to +\infty \quad g(x) \to +\infty$