Math 3 Review Sheet Unit 1

(SPRING 2017)

① @ $\begin{cases} 2x-4y=6 \\ x+2y=3 \end{cases}$

*different slopes

ONE SOLUTION

(can put into $y=mx+b$ form to more easily see slope)

⑥ $\begin{cases} 2x-4y=6 \\ x-2y=3 \end{cases}$

*same slope, same y-intercept

INFINITE # OF SOLUTIONS

© $\begin{cases} 2x+4y=6 \\ x+2y=6 \end{cases}$

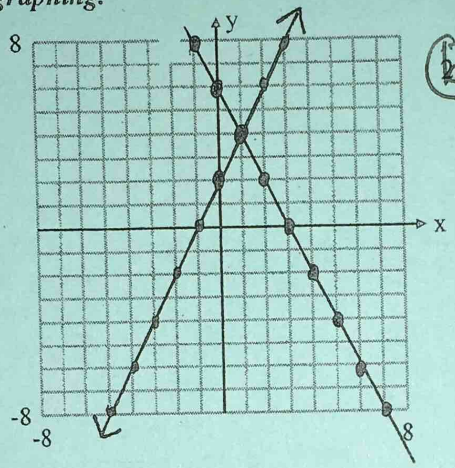
*same slope, diff. y-intercept

NO SOLUTION $\emptyset$

Solve these equations by graphing.

② @ $\begin{cases} y=2x+2 \\ y+2x=6 \end{cases}$

$y=-2x+6$



(1, 4)

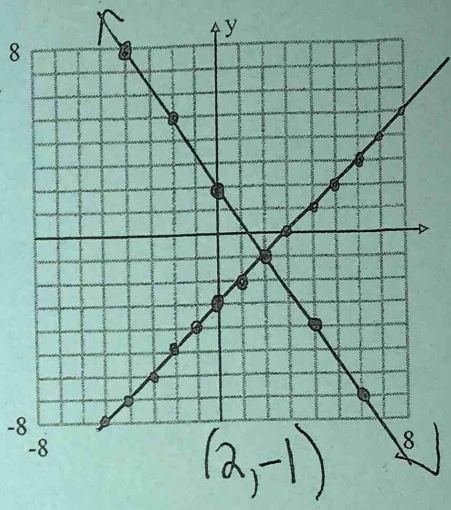
⑥ $\begin{cases} 3x+2y=4 \\ x-y=3 \end{cases}$

$2y=-3x+4$

$y=\frac{-3}{2}x+2$

$-y=-x+3$

$y=x-3$



(2, -1)

③ $\begin{cases} y=x-3 \\ 4x+y=32 \end{cases}$ substitution → "circle & stick"

$4x+(x-3)=32$

$4x+x-3=32$

$5x=35$

$x=7$

now solve for y using the circle

$y=7-3$

$y=4$

(7, 4)

④ $\begin{cases} 5x+2y=7 \\ 3x-y=13 \text{ (mult. 2)} \end{cases}$

elimination → make 1 variable go away

I'm going to eliminate y

$+\begin{cases} 5x+2y=7 \\ 6x-2y=26 \end{cases}$

$11x=33$

$x=3$

now solve for y using either original equation

plug into bottom eqn:

$3(3)-y=13$

$9-y=13$

$-y=4$

$y=-4$

(3, -4)

⑤ $\begin{cases} y=x+3 \\ 4x+y=8 \end{cases}$ → $y=1+3$

$y=4$

$4x+(x+3)=8$

$4x+x+3=8$

$5x=5$

$x=1$

(1, 4)

⑥ $\begin{cases} 2x+3y=4 \text{ (mult. 2)} \\ -4x-6y=-8 \end{cases}$

$\begin{cases} 4x+6y=8 \\ -4x-6y=-8 \end{cases}$

$0=0$ ← *this is TRUE!

$\{(x,y) \mid 2x+3y=4\}$

(7) $\begin{cases} x+3y=-2 \\ x-2y=-3 \end{cases}$ → $x=-2-3y$

$(-2-3y)-2y=-3$

$-2-3y-2y=-3$

$-2-5y=-3$

$-5y=-1$

$y=\frac{1}{5}$

$x=-2-3(\frac{1}{5})$

$x=-2-\frac{3}{5}$

$x=-\frac{13}{5}$

$\left(-\frac{13}{5}, \frac{1}{5}\right)$

(8) $\begin{cases} 4x-2y=5 \\ 2x=y-1 \end{cases}$ → $y=2x+1$

$4x-2(2x+1)=5$

$4x-4x-2=5$

$-2=5$ ← this is FALSE!

$\emptyset$

(9) $\begin{cases} 2x+3y+z=-5 \\ x-y-3z=-8 \\ 3x+2y+z=-4 \end{cases}$ eliminate z

① $2x+3y+z=-5$ (mult. 3)

② $x-y-3z=-8$

$\begin{cases} 6x+9y+3z=-15 \\ +\ x-y-3z=-8 \end{cases}$

$7x+8y=-23$ ★

① $2x+3y+z=-5$

③ $3x+2y+z=-4$ (mult. -1)

$\begin{cases} 2x+3y+z=-5 \\ +\ -3x-2y-z=4 \end{cases}$

$-x+y=-1$ ★

★ $\begin{cases} 7x+8y=-23 \\ -x+y=-1 \end{cases}$ → $y=x-1$

$7x+8(x-1)=-23$

$7x+8x-8=-23$

$15x=-15$

$x=-1$

$y=-1-1$

$y=-2$

Solve for z using equation 1: $2(-1)+3(-2)+z=-5$

$-2-6+z=-5$

$-8+z=-5$

$z=3$

$(-1,-2,3)$

(10) $\begin{cases} 2x-3y+z=-19 \\ 3x+4y-2z=21 \\ x-2y+z=-13 \end{cases}$ eliminate z

① $2x-3y+z=-19$ (mult. 2)

② $3x+4y-2z=21$

$\begin{cases} 4x-6y+2z=-38 \\ +\ 3x+4y-2z=21 \end{cases}$

$7x-2y=-17$ ★

① $2x-3y+z=-19$

③ $x-2y+z=-13$ (mult. -1)

$\begin{cases} 2x-3y+z=-19 \\ +\ -x+2y-z=13 \end{cases}$

$x-y=-6$ ★

★ $\begin{cases} 7x-2y=-17 \\ x-y=-6 \text{ (mult. } -7) \end{cases}$ eliminate x

$\begin{cases} 7x-2y=-17 \\ +\ -7x+7y=42 \end{cases}$

$5y=25$

$y=5$

Solve for x using bottom ★ equation: $x-5=-6$

$x=-1$

Solve for z using equation 3: $(-1)-2(5)+z=-13$

$-11+z=-13$

$z=-2$

$(-1,5,-2)$

(11) a = # of adults c = # of children

$\begin{cases} 3c+4.5a=975 \\ c+a=250 \end{cases}$ → $c=250-a$

$3(250-a)+4.5a=975$

$750-3a+4.5a=975$

$1.5a=225$

$a=150$

$c=250-150$

$c=100$

150 adults

100 children

x = price sandwich
y = price soup

$$\begin{cases} 2x+y=2.75 \\ 3x+2y=4.75 \end{cases}$$ (mult. -2)

$$+\begin{cases} -4x-2y=-5.50 \\ 3x+2y=4.75 \end{cases}$$

$-x = -.75$

$x = .75$

Solve for y using top equation:

$2(.75)+y=2.75$

$1.5+y=2.75$

$y=1.25$

Sandwiches cost 75¢, Soup costs $1.25

(13) X = $ invested at 6% y = $ invested at 9% z = $ invested at 11%

$$\begin{cases} x+y+z=17{,}000 \\ .06x+.09y+.11z=1540 \\ 3x=y \end{cases}$$

rewrite →

$$\begin{cases} ① \; x+y+z=17{,}000 \\ ② \; .06x+.09y+.11z=1540 \\ ③ \; 3x-y=0 \end{cases}$$

*since z is already missing from equation 3, that is the best variable to eliminate!

③ $3x-y=0$ ★

this doesn't have a z so it's automatically a ★ equation!

$$\begin{cases} ① \; x+y+z=17{,}000 \text{ (mult. } -.11) \\ ② \; .06x+.09y+.11z=1540 \end{cases}$$

$$+\begin{cases} -.11x-.11y-.11z=-1870 \\ .06x+.09y+.11z=1540 \end{cases}$$

$-.05x-.02y = -330$ ★

$$\star\begin{cases} 3x-y=0 \rightarrow -y=-3x \\ .05x+.02y=-330 \end{cases}$$

$y=3x$

$y=3(3000)$

$y=9000$

$-.05x-.02(3x)=-330$

$-.05x-.06x=-330$

$-.11x=-330$

$x=3000$

Solve for z using equation 1:

$3000+9000+z=17000$

$12000+z=17000$

$z=5000$

$3000 at 6%
$9000 at 9%
$5000 at 11%

(14) n = # of nickels d = # of dimes q = # quarters

$$\begin{cases} n+d+q=25 \\ .05n+.10d+.25q=4.90 \\ d=2n-1 \end{cases}$$

rewrite →

$$\begin{cases} ① \; n+d+q=25 \\ ② \; .05n+.10d+.25q=4.90 \\ ③ \; -2n+d=-1 \end{cases}$$

*since q is already missing from equation 3, it is the easiest variable to eliminate!

③ $-2n+d=-1$ ★

$$\begin{cases} ① \; n+d+q=25 \text{ (mult. } -.25) \\ ② \; .05n+.10d+.25q=4.90 \end{cases}$$

$$+\begin{cases} -.25n-.25d-.25q=-6.25 \\ .05n+.10d+.25q=4.90 \end{cases}$$

$-.2n+.15d = -1.35$ ★

$$\star\begin{cases} -2n+d=-1 \rightarrow d=2n-1 \\ -.2n+.15d=-1.35 \end{cases}$$

$d=2(3)-1$

$d=6-1$

$d=5$

$-.2n+.15(2n-1)=-1.35$

$-.2n+.3n+.15=-1.35$

$-.5n=-1.5$

$n=3$

Solve for q using equation 1:

$3+5+q=25$

$8+q=25$

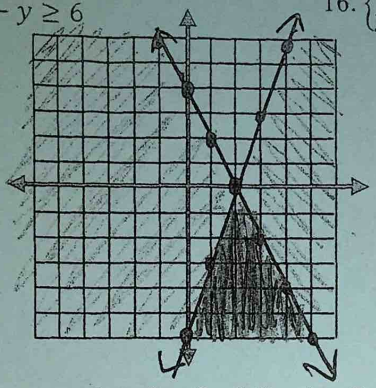
$q=17$

3 nickels
5 dimes
17 quarters

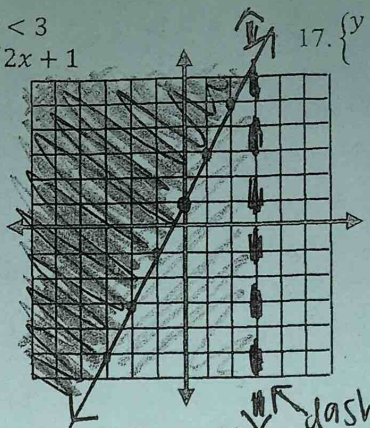
15. $\begin{cases} 2x + y \le 4 \\ 3x - y \ge 6 \end{cases}$

$-y \ge -3x+6$

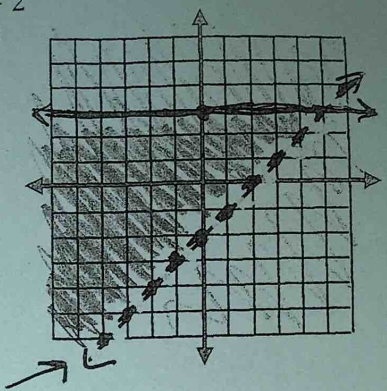
$y \le 3x-6$



16. $\begin{cases} x < 3 \\ y \ge 2x+1 \end{cases}$



17. $\begin{cases} y > x-2 \\ y \le 3 \end{cases}$



dashed

Write the constraints for the following linear programming problem:

18. The area of a parking lot is 600 square meters. A car requires 6 square meters. A bus requires 30 square meters. The attendant can handle only 60 vehicles. If a car is charged \$2.50 and a bus \$7.50, how many of each should be accepted to maximize income?

c = # cars, b = # buses

$P = 2.50c + 7.50b$

$\begin{cases} 6c+30b \le 600 \\ c+b \le 60 \\ c \ge 0 \\ b \ge 0 \end{cases}$

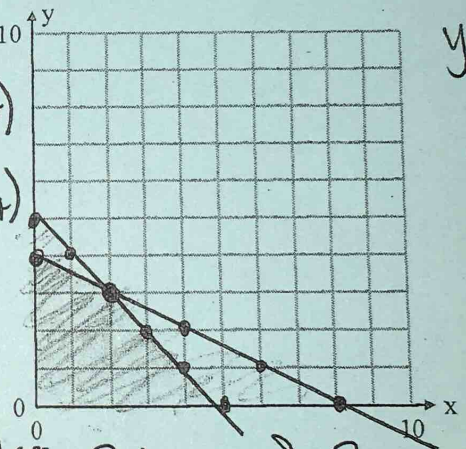
19. Baking a tray of corn muffins takes 4 cups of milk and 3 cups of flour. A tray of bran muffins takes 2 cups of milk and 3 cups of flour. A baker has 16 cups of milk and 15 cups of flour. He makes \$3 profit per tray of corn muffins and \$2 profit per tray of bran muffins. How many trays of each type of muffin should the baker make to maximize his profit?

Let x = the number of trays of bran muffins
Let y = the number of trays of corn muffins

Constraints: $\begin{cases} 4y+2x \le 16 & \text{(milk constraint)} \\ 3y+3x \le 15 & \text{(flour constraint)} \\ y \ge 0 \\ x \ge 0 \end{cases}$

$y \le -\frac{1}{2}x+4$

$y \le -x+5$



Profit Equation: $3y+2x=P \rightarrow P=2x+3y$

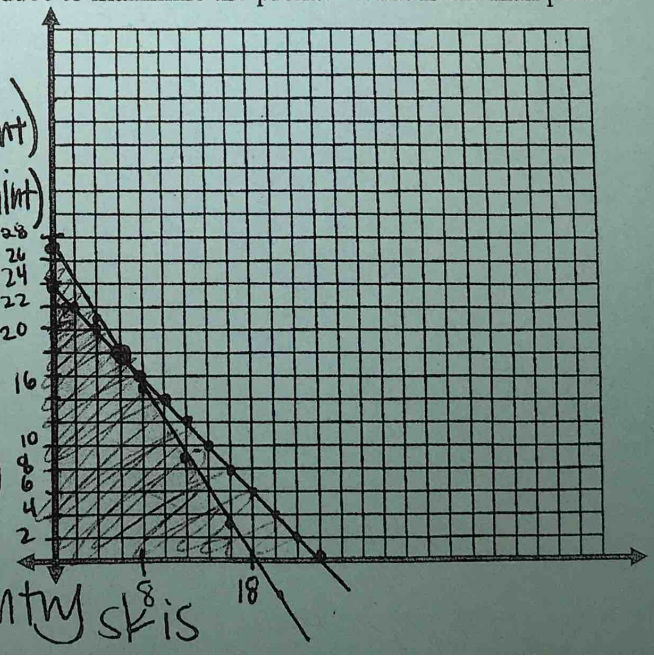
Corner Points: (0,0) (0,4) (2,3) (5,0)

Solution: Max. profit is \$13 when he makes 2 bran & 3 corn muffins

20. A ski company makes two types of skis and has a fabrication and a finishing department. A pair of downhill skis requires 6 hours to fabricate and 1 hour to finish. A pair of cross-country skis requires 4 hours to fabricate and 1 hour to finish. The fabricating department has 108 hours of labor available per day. The finishing department has 24 hours of labor available per day. The company makes a profit of \$40 on each pair of downhill skis and \$30 on each pair of cross-country skis. How many of each type should the manufacturer produce to maximize the profit? What is the max profit?

Variables: x = # downhill skis
y = # cross-country skis

Constraints: $\begin{cases} 6x+4y \le 108 & \text{(fabricating constraint)} \\ x+y \le 24 & \text{(finishing constraint)} \\ x \ge 0 \\ y \ge 0 \end{cases}$



Profit Equation: $P = 40x+30y$

Corner Points: (0,0) (0,24) (6,18) (18,0)

Solution: Max profit \$800

6 downhill skis, 18 cross-country skis