**Honors Math 3**

**Direct, Joint, and Inverse Variation**

When two variable quantities have a constant (unchanged) ratio, their relationship is called a **direct variation.** It is said that one variable "varies directly" as the other. The constant ratio is called the constant of variation. The formula for direct variation is **y = kx,** where k is called the **constant of variation.** In a direct variation, the two variables change in the same sense. If one increases, so does the other.

For example, let’s say that the weekly salary a woman earns, S, varies directly as the number of hours, h, which she works. Algebraically, we can say that .

If the woman earns $116.25 for working 15 hours, what is the constant of variation?

Graphically, 7.75 would be the slope of the salary function for this particular situation.

Now find the amount the woman will earn if she works 23.5 hours:

In an **inverse variation**, the values of the two variables change in an opposite manner - as one value increases, the other decreases. For instance, a biker traveling at 8 mph can cover 8 miles in 1 hour. If the biker's speed decreases to 4 mph, it will take the biker 2 hours (an increase of one hour), to cover the same distance. An inverse variation between 2 variables, y and x, is a relationship that is expressed as:

where the variable k is called the **constant of proportionality**. As with the direct variation problems, the k value needs to be found using the first set of data.

For example, in kick boxing, it is found that the force, f, needed to break a board, varies inversely with the length, l, of the board. If it takes 5 lbs of pressure to break a board 2 feet long, how many pounds of pressure will it take to break a board that is 6 feet long?

now use 10 as your k value:

f

f = 1.67 lbs of pressure

When we say z is **jointly proportional** to a set of variables, it means that z is directly proportional to each variable taken one at a time. If z varies jointly with x and y, the equation will be of the form **z = kxy** (where k is a constant). Joint variation can also combine direct and inverse variations. For example, if z varies jointly with x and y and inversely with the square root of w, the equation will be: .

For example, the volume of a pyramid varies jointly as its height and the area of its base. A pyramid with a height of 12 feet and a base with area of 23 square feet has a volume of 92 cubic feet. Find the volume of a pyramid with a height of 17 feet and a base with an area of 27 square feet.

V V = 1/3*hA*

92 V = 1/3(17)(27)

1/3 V = 153 cubic feet

***Please make sure your answers are labeled with the correct units!***

***For each problem, find the constant of variation, write an equation, and solve.***

1. The number of gallons of fuel used on a trip varies directly with the number of miles traveled. If a trip of 270 miles required 12 gallons of fuel, how many gallons are required for a trip of 400 miles?

2. Wei received $55.35 in interest on the $1230 in her credit union account. If the interest varies directly with the amount deposited, how much would Wei receive for the same amount of time if she had $2000 in the account?

3. The cost of materials for a deck varies jointly with the width and the length. If the cost is $470.40 when w = 12 and l = 16, find the cost when w = 10 and l = 25.

4. The value of real estate V varies jointly with the neighborhood index N and the square footage of the house S. If V = $376, 320 when N = 96 and S = 1600, find the value of a property with N = 83 and S = 2150.

5. The number of gallons in a circular swimming pool varies jointly with the square of the radius and the depth. If there are 754 gallons when the radius is 4 and the depth is 2, find the number of gallons in the pool when the radius is 3 and the depth is 1.5.

6. The volume V of a gas kept at a constant temperature varies inversely as the pressure p. If the pressure is 24 pounds per square inch, the volume is 15 cubic feet. What will be the volume when the pressure is 30 pounds per square inch?

7. The time to complete a project varies inversely with the number of employees. If 3 people can complete the project in 7 days, how long will it take 5 people?

8. The time needed to travel a certain distance varies inversely with the rate of speed. If it takes 8 hours to travel a certain distance at 36 miles per hour, how long will it take to travel the same distance at 60 miles per hour?

9. The number of revolutions made by a tire traveling over a fixed distance varies inversely to the radius of the tire. A 12-inch radius tire makes 100 revolutions to travel a certain distance. How many revolutions would a 16-inch radius tire require to travel the same distance?

10. To build a sound wall along the highway, the amount of time t needed varies directly with the number of cement blocks c needed and inversely with the number of workers w. A sound wall made of 2400 blocks, using six workers takes 18 hours to complete. How long would it take to build a wall of 4500 blocks with 10 workers?

11. The time needed to paint a fence varies directly with the length of the fence and inversely with the number of painters. If it takes five hours to paint 200 feet of fence with three painters, how long will it take five painters to paint 500 feet of fence?

12. The time to prepare a field for planting is inversely proportional to number of people who are working. A large field can be prepared by five workers in 24 days. In order to finish the field sooner, the farmer plans to hire additional workers. How many workers are needed to finish the field in 15 days?

13. An egg is dropped from the roof of a building. The distance it falls varies directly with the square of the time it falls. If it takes 12 second for the egg to fall eight feet, how long will it take the egg to fall 200 feet?

14. The number of hours needed to assemble computers varies directly as the number of computers and inversely as the number of workers. If 4 workers can assemble 12 computers in 9 hours, how many workers are needed to assemble 48 computers in 8 hours?

15. The weight of a person varies inversely as the square of the distance from the center of the earth. If the radius of the earth is 4000 miles, how much would a 180 pound person weigh, 2000 miles above the surface of the earth?

16. The strength of a rectangular beam varies jointly as its width and the square of its depth. If the strength of a beam three inches wide by 10 inches deep is 1200 pounds per square inch, what is the strength of a beam four inches wide and six inches deep?