AFM Arithmetic Series WS

① $3+5+7+9+\ldots+57$

$S_n = ?$ $a_1 = 3$ $d = 2$ $a_n = 57$

Series: $S_n = \frac{n}{2}(3+57)$

go back to sequence to find n

sequence: $57 = 3 + (n-1)(2)$

$54 = (n-1)(2)$

$27 = n-1$

$28 = n$

$S_{28} = \frac{28}{2}(3+57)$

$\boxed{S_{28} = 840}$

② $1-3-7-11\ldots-51$

$S_n = ?$ $a_1 = 1$ $d = -4$ $a_n = -51$

Series: $S_n = \frac{n}{2}(1-51)$

sequence: $-51 = 1 + (n-1)(-4)$

$-52 = (n-1)(-4)$

$13 = n-1$

$14 = n$

$S_{14} = \frac{14}{2}(1-51)$

$\boxed{S_{14} = -350}$

③ $5-2-9-16\ldots-156$

$S_n = ?$ $a_1 = 5$ $d = -7$ $a_n = -156$

Series: $S_n = \frac{n}{2}(5-156)$

Sequence: $-156 = 5 + (n-1)(-7)$

$-161 = (n-1)(-7)$

$23 = n-1$

$24 = n$

$S_{24} = \frac{24}{2}(5-156)$

$\boxed{S_{24} = -1812}$

④ $a_1 = -12$ $a_{14} = 54$ $S_{14} = ?$

$S_{14} = \frac{14}{2}(-12+54)$

$\boxed{S_{14} = 294}$

⑤ $S_{48} = ?$ $a_1 = -6$ $d = 2$

series: $S_{48} = \frac{48}{2}(-6 + a_{48})$

sequence: $a_{48} = -6 + (48-1)(2)$

$a_{48} = 88$

$S_{48} = \frac{48}{2}(-6+88)$

$\boxed{S_{48} = 1968}$

(6) $a_1 = -11$ $d = 4$ $a_n = 137$

series: $S_n = n/2(-11 + 137)$

sequence: $137 = -11 + (n-1)(4)$

$148 = (n-1)(4)$

$37 = n-1$

$38 = n$

$S_{38} = 38/2(-11+137)$

$\boxed{S_{38} = 2394}$

(7) $S_{16} = ?$ $-18, -15, -12 \ldots$

$a_1 = -18$ $d = 3$

series: $S_{16} = 16/2(-18 + a_{16})$

sequence: $a_{16} = -18 + (16-1)(3)$

$a_{16} = 27$

$S_{16} = 16/2(-18+27)$

$\boxed{S_{16} = 72}$

(8) $S_n = ?$ $22, 25, 28 \ldots 73$

$a_1 = 22$ $d = 3$ $a_n = 73$

series: $S_n = n/2(22+73)$

sequence: $73 = 22 + (n-1)(3)$

$51 = (n-1)(3)$

$17 = n-1$

$18 = n$

$S_{18} = 18/2(22+73)$

$\boxed{S_{18} = 855}$

(9) $-29, -25, -21, \ldots 91$

$a_1 = -29$ $d = 4$ $a_n = 91$

series: $S_n = n/2(-29+91)$

sequence: $91 = -29 + (n-1)(4)$

$120 = (n-1)(4)$

$30 = n-1$

$31 = n$

$S_{31} = 31/2(-29+91)$

$\boxed{S_{31} = 961}$

(10) $(-10)+(-7)+(-4)+\ldots \quad S_n=200$

Series: $200 = \frac{n}{2}(-10+a_n)$

Sequence: $a_n = -10+(n-1)(3)$

$a_n = -10+3n-3$

$a_n = 3n-13$

$200 = \frac{n}{2}(-10+3n-13)$

$400 = n(-23+3n)$

$400 = -23n+3n^2$

$0 = 3n^2-23n-400$

★graph in calc★

$\boxed{n=16}$

(11) $25+19+13+\ldots \quad S_n=-20$

Series: $-20 = \frac{n}{2}(25+a_n)$

sequence: $a_n = 25+(n-1)(-6)$

$a_n = 25-6n+6$

$a_n = 31-6n$

$-20 = \frac{n}{2}(25+31-6n)$

$-40 = n(56-6n)$

$-40 = 56n-6n^2$

$6n^2-56n-40=0$

★graph in calc★

$\boxed{n=10}$

(12) $95+90+85+\ldots \quad S_n=0$

series: $0 = \frac{n}{2}(95+a_n)$

sequence: $a_n = 95+(n-1)(-5)$

$a_n = 95-5n+5$

$a_n = 100-5n$

$0 = \frac{n}{2}(95+100-5n)$

$0 = n(195-5n)$

$0 = 195n-5n^2$

★Graph in calc★

$\boxed{n=39}$

(13) $a_5=16 \quad a_{11}=4 \quad S_{20}=?$

$S_{20} = \frac{20}{2}(a_1+a_{20})$

• manipulate eqn: $a_n = 16+(n-5)d$

• find d: $4 = 16+(11-5)d$

$-12 = 6d$

$-2 = d$

• update eqn: $a_n = 16+(n-5)(-2)$

• find a_1: $a_1 = 16+(1-5)(-2)$

$a_1 = 24$

• find a_{20}: $a_{20} = 16+(20-5)(-2)$

$a_{20} = -14$

$S_{20} = \frac{20}{2}(24-14)$

$\boxed{S_{20}=100}$

(14) $a_7 = 6 \quad a_{13} = 24 \quad S_{15} = ?$

$S_{15} = 15/2(a_1 + a_{15})$

- manipulate eqn: $a_n = 6 + (n-7)d$
- find d: $24 = 6 + (13-7)d$

$18 = 6d$

$3 = d$

- update eqn: $a_n = 6 + (n-7)(3)$
- find a_1: $a_1 = 6 + (1-7)(3)$

$a_1 = -12$

- find a_{15}: $a_{15} = 6 + (15-7)(3)$

$a_{15} = 30$

$S_{15} = 15/2(-12 + 30)$ $\boxed{S_{15} = 135}$

(15) $a_{13} = 7 \quad a_{17} = 23 \quad S_{20} = ?$

$S_{20} = 20/2(a_1 + a_{20})$

- manipulate eqn: $a_n = 7 + (n-13)d$
- find d: $23 = 7 + (17-13)d$

$16 = 4d$

$d = 4$

- update eqn: $a_n = 7 + (n-13)(4)$
- find a_1: $a_1 = 7 + (1-13)(4)$

$a_1 = -41$

- find a_{20}: $a_{20} = 7 + (20-13)(4)$

$a_{20} = 35$

$S_{20} = 20/2(-41 + 35)$ $\boxed{S_{20} = -60}$

(16) $a_{10} = 17x - 11y \quad a_{15} = 7x - 6y \quad S_{25} = ?$

$S_{25} = 25/2(a_1 + a_{25})$

- manipulate eqn: $a_n = 17x - 11y + (n-10)d$
- find d: $7x - 6y = 17x - 11y + (15-10)d$

$-10x + 5y = 5d$

$-2x + y = d$

- update eqn: $a_n = 17x - 11y + (n-10)(-2x+y)$
- find a_1: $a_1 = 17x - 11y + (1-10)(-2x+y)$

$a_1 = 17x - 11y + 18x - 9y$

$a_1 = 35x - 20y$

- find a_{25}: $a_{25} = 17x - 11y + (25-10)(-2x+y)$

$a_{25} = 17x - 11y - 30x + 15y$

$a_{25} = -13x + 4y$

- find S_{25}:

$S_{25} = 25/2(35x - 20y - 13x + 4y)$

$S_{25} = 25/2(22x - 16y)$

$\boxed{S_{25} = 275x - 200y}$

(17) sum of integers 2–51

$S_n = n/2(2+51)$

sequence: $51 = 2+(n-1)(1)$

$49 = n-1$

$50 = n$

$S_{50} = 50/2(2+51)$

$S_{50} = 1325$

(18) integers −59 – −16

$S_n = n/2(-59+-16)$

sequence: $-16 = -59+(n-1)(1)$

$43 = n-1$

$44 = n$

$S_{44} = 44/2(-59-16)$

$S_{44} = -1650$

(19) even integers 38–62

38, 40, 42, ... 62

$S_n = n/2(38+62)$

sequence: $62 = 38+(n-1)(2)$

$24 = (n-1)(2)$

$12 = n-1$

$13 = n$

$S_{13} = 13/2(38+62)$

$S_{13} = 650$

(20) odd integers 19 through 41

19, 21, 23, ... 41

$S_n = n/2(19+41)$

sequence: $41 = 19+(n-1)(2)$

$22 = (n-1)(2)$

$11 = n-1$

$12 = n$

$S_{12} = 12/2(19+41)$

$S_{12} = 360$

(21) positive multiples of 3, less than 100

3, 6, 9, ... 99

$S_n = n/2(3+99)$

sequence: $99 = 3+(n-1)(3)$

$96 = (n-1)(3)$

$32 = n-1$

$33 = n$

$S_{33} = 33/2(3+99)$ $S_{33} = 1683$

(22) multiples of 5, less than 80

5, 10, 15 75

$S_n = n/2(5+75)$

sequence: $75 = 5+(n-1)(5)$

$70 = (n-1)(5)$

$14 = n-1$

$15 = n$

$S_{15} = 15/2(5+75)$ $S_{15} = 600$

(23) even integers between 37 & 81

~~37~~ 38, 40, 42, ... 80

$S_n = n/2\,(38+80)$

Sequence: $80 = 38 + (n-1)(2)$

$42 = (n-1)(2)$

$21 = n-1$

$22 = n$

$S_{22} = 22/2\,(38+80)$ $\boxed{S_{22} = 1298}$

(24) odd integers 15 to 161 inclusive

15, 17, 19, ... 161

$S_n = n/2\,(15+161)$

Sequence: $161 = 15 + (n-1)(2)$

$146 = (n-1)(2)$

$73 = n-1$

$74 = n$

$S_{74} = 74/2\,(15+161)$

$\boxed{S_{74} = 6512}$