AFM UNIT 5 – COMBINATORICS REVIEW Name key

Determine whether each is a permutation (P) or Combination (C)

1. Arranging 7 different books on a shelf P
2. Assigning seats in a classroom P
3. Selecting a class committee of 4 students C
4. Choosing class officers (pres, VP, etc.) P
5. Getting a poker hand of 5 cards C
6. Seating 8 guests around a dinner table P

Determine if the following permutations are linear, circular, or reflective.

7. Nine people at a round table relative to a door L
8. 6 players in a huddle C
9. Charms on a necklace with no clasp C, R
10. Arranging digits of a 5 digit number L
11. Keys on a key ring C, R
12. flags strung on a flagpole L, R

Calculate the following using permutations, combinations, or the counting principle. SHOW YOUR WORK!

13. Seven different letters form a 4-letter pin code. If the letters **can be repeated**, how many different codes are possible?

$7 \cdot 7 \cdot 7 \cdot 7 = 2401$

14. A restaurant serves 5 main dishes, 4 salads, and 8 desserts. How many different meals can be ordered if each has a main dish, a salad, and a dessert?

$5 \cdot 4 \cdot 8 = 160$

15. In how many ways can 7 members of a family be seated side by side in a movie theater if Dad has to sit at the one and only aisle seat?

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Dad $6! = 720$

16. There are 8 different Taylor Swift songs on iTunes. You have enough money to buy 4 of them. How many different selections are possible?

$C(8,4) = \frac{8!}{(4!4!)} = 70$

17. An exam has 5 essay questions. You are required to answer 4 of them. How many different selections are possible?

$C(5,4) = \frac{5!}{(1!4!)} = 5$

18. How many ways can you arrange 6 math books, 2 social studies cooks, and 2 science books on a shelf if the subjects must stay together?

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot \underbrace{2 \cdot 1}_{2} \cdot \underbrace{2 \cdot 1}_{1}$ (3)

$6!2!2!3! = 17280$

19. Four boys and 4 girls are lining up for a photo. How many arrangements are possible if they want to alternate by gender?

anyone can sit down first → $8 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 1152$

20. How many ways can 6 keys be arranged on a key ring? $\frac{(6-1)!}{2} = 60$

circular, reflective

21. Your school yearbook has an editor-in-chief position and an assistant editor-in-chief. There are 15 students on yearbook staff. How many ways can 2 students be chosen for these positions?

$15 \cdot 14$ or $P(15,2) = \frac{15!}{13!} = 210$

22. How many ways can the word "HIPPOPOTAMUS" be arranged? 12 total

P → 3
O → 2

$\frac{12!}{(3!2!)} = 39{,}916{,}800$

23. How many different committees can be formed from 6 teachers and 50 students if the committee must have 2 teachers and 3 students?

$C(6,2) \cdot C(50,3) = \frac{6!}{(4!2!)} \cdot \frac{50!}{(47!3!)} = 15 \cdot 19600 = 294{,}000$

24. Ms. Norris and her twin are taking a picture with 6 of their friends on their birthday. How many arrangements are possible if Ms. Norris and her twin have to be in the middle?

$6 \cdot 5 \cdot 4 \cdot \underset{\text{Ms. Norris}}{2} \cdot \underset{\text{Twin}}{1} \cdot 3 \cdot 2 \cdot 1 = 6!2! = 1440$

25. Expand $(2x-3)^4$ using the binomial expansion formula.

$C(4,0)(2x)^4(-3)^0 + C(4,1)(2x)^3(-3)^1 + C(4,2)(2x)^2(-3)^2 + C(4,3)(2x)^1(-3)^3 + C(4,4)(2x)^0(-3)^4$

$1 \cdot 16x^4 + 4 \cdot 8x^3 \cdot -3 + 6 \cdot 4x^2 \cdot 9 + 4 \cdot 2x \cdot -27 + 1 \cdot 81$

$16x^4 - 96x^3 + 216x^2 - 216x + 81$

26. Find the 4th term of the expansion of $(a-2b)^7$.

$C(7,3)a^4(-2b)^3 \quad 35 \cdot a^4 \cdot -8b^3 \quad -280a^4b^3$

27. Find the coefficient of the 5th term of the expansion of $(x-2)^8$.

$C(8,4)x^4(-2)^4 \quad 70 \cdot x^4 \cdot 16$ coefficient: 1120

Solve the following for n:

28. $\frac{n!}{(n-2)!} = 12$

$\frac{n \cdot (n-1) \cdot (n-2)!}{(n-2)!} = 12$

$n^2 - n = 12$
$n^2 - n - 12 = 0$
$(n-4)(n+3) = 0$

$n = 4$ $\quad n = -3$

29. $\frac{(n-3)!}{(n-5)!} = 42$

$\frac{(n-3)(n-4)(n-5)!}{(n-5)!} = 42$

$n^2 - 7n + 12 = 42$
$n^2 - 7n - 30 = 0$
$(n-10)(n+3) = 0$

$n = 10$ $\quad n = -3$

30. $4!\,n! = 5!\,(n-1)!$

$4! \cdot n \cdot (n-1)! = 5!(n-1)!$

$4!\,n = \frac{5!(n-1)!}{(n-1)!}$

$4!\,n = 5!$
$n = \frac{5!}{4!}$

$n = 5$

Simplify the following:

31. $\frac{8!}{5!3!}$

$\frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 3 \cdot 2 \cdot 1}$

56

32. $\frac{5!}{3!} + \frac{9!}{7!}$

$\frac{5 \cdot 4 \cdot 3!}{3!} + \frac{9 \cdot 8 \cdot 7!}{7!}$

$20 + 72$

92

33. $(n+1)!\,(n+2)$

$(n+2)!$