AFM Matrices Review

① $2\begin{bmatrix} 2 & -1 & 4 \\ -3 & 2 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -5 \\ 2 & 4 & 7 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 2 \\ 3 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 4 & -2 & 8 \\ -6 & 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 3 & -7 \\ -1 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 1 \\ -7 & 8 & 2 \end{bmatrix}$

② $3\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & 0 \end{bmatrix}$

$\begin{bmatrix} -6 & -3 & 15 \\ -3 & 6 & 0 \end{bmatrix}$

③ $-\frac{1}{2}\begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix} + 2\begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix} + \begin{bmatrix} -6 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} -8 \\ 5 \\ -4 \end{bmatrix}$

④ $\begin{bmatrix} 3 & 1 \\ 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$

rows x columns

row 1 x column 1 : $3 \cdot 1 + 1 \cdot 2 = 3 + 2$

row 1 x column 2 : $3 \cdot -1 + 1 \cdot 0 = -3 + 0$

row 2 x column 1 : $2 \cdot 1 + -2 \cdot 2 = 2 - 4$

row 2 x column 2 : $2 \cdot -1 + -2 \cdot 0 = -2 + 0$

$\begin{bmatrix} 5 & -3 \\ -2 & -2 \end{bmatrix}$

⑤ $\begin{bmatrix} -2 & 4 & 5 \\ 3 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 0 & 2 & -5 \end{bmatrix}$

UNDEFINED

⑥ $\begin{bmatrix} 2 & -1 & 3 \\ 2 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 9 & -3 \\ 2 & -5 \end{bmatrix}$

3x3 2x3

2x3 3x2

$= \begin{bmatrix} -1 & -12 \\ 38 & -12 \end{bmatrix}$

row 1 x column 1 : $2 \cdot 1 + -1 \cdot 9 + 3 \cdot 2 = 2 - 9 + 6$

row 1 x column 2 : $2 \cdot 0 + -1 \cdot -3 + 3 \cdot -5 = 0 + 3 - 15$

row 2 x column 1 : $2 \cdot 1 + 4 \cdot 9 + 0 \cdot 2 = 2 + 36 + 0$

row 2 x column 2 : $2 \cdot 0 + 4 \cdot -3 + 0 \cdot -5 = 0 - 12 + 0$

(7) $\begin{bmatrix} 4x & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -3 \\ x & -2y \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 0 & -3 \end{bmatrix}$

row 1 × column 1 = −2

$4x \cdot 0 + 2 \cdot x = -2$

$2x = -2$

$x = -1$

row 1 × column 2 = 4

$4x \cdot -3 + 2 \cdot -2y = 4$

$4 \cdot -1 \cdot -3 - 4y = 4$

$12 - 4y = 4$

$-4y = -8$

$y = 2$

(8) $\begin{bmatrix} -1 & 5 \\ 3 & 4 \end{bmatrix} + 5\begin{bmatrix} 0 & 5 \\ -2 & -3 \end{bmatrix}$

$\begin{bmatrix} -1 & 5 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 25 \\ -10 & -15 \end{bmatrix}$

$\begin{bmatrix} -1 & 30 \\ -7 & -11 \end{bmatrix}$

(9) $\begin{vmatrix} -2 & 10 \\ -1 & 4 \end{vmatrix}$ $-2 \cdot 4 - 10 \cdot -1$

$-8 + 10$

2

(10) $\begin{vmatrix} -4 & 0 & -1 \\ -5 & -3 & 2 \\ -2 & -1 & 2 \end{vmatrix}$ $\begin{matrix} -4 & 0 \\ -5 & -3 \\ -2 & -1 \end{matrix}$

Can use Diagonals or Expansion by minors

$(-4 \cdot -3 \cdot 2) + (0 \cdot 2 \cdot -2) + (-1 \cdot -5 \cdot -1) - (0 \cdot -5 \cdot 2) - (-4 \cdot 2 \cdot -1) - (-1 \cdot -3 \cdot -2)$

$24 + 0 \quad -5 \quad -0 \quad -8 \quad +6$

17

(11) $A = \begin{bmatrix} 7 & 1 & -6 & 0 & -3 \\ 2 & 1 & 0 & -5 & -4 \\ 1 & 3 & -7 & -9 & 9 \end{bmatrix}$

Dimensions: 3×5 $\quad a_{13}: -6$ $\quad a_{34}: -9$

$a_{21}: 2$ $\quad a_{31}: 1$

(12) $\begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix}^{-1}$ *start by finding the determinant!

$-1 \cdot 3 - (-2 \cdot 1)$

$-3 - (-2)$

$-3 + 2$

-1

$\frac{1}{-1}\begin{bmatrix} 3 & 2 \\ -1 & -1 \end{bmatrix}$

$\begin{bmatrix} -3 & -2 \\ 1 & 1 \end{bmatrix}$

(13) $\begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}^{-1}$ $-3 \cdot 4 - 6 \cdot -2$

$-12 - (-12)$

$-12 + 12$

0

*no inverse exists!

(14) $X + \begin{bmatrix} -7 & 0 \\ 4 & -1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 6 & 10 \\ 8 & -3 \\ -1 & -9 \end{bmatrix}$

$X = \begin{bmatrix} 6 & 10 \\ 8 & -3 \\ -1 & -9 \end{bmatrix} - \begin{bmatrix} -7 & 0 \\ 4 & -1 \\ 2 & -3 \end{bmatrix}$

$X = \begin{bmatrix} 13 & 10 \\ 4 & -2 \\ -3 & -6 \end{bmatrix}$

(15) $\begin{cases} x + y = 19 \\ 4x + 2y = -8 \end{cases}$

• denominator: $\begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix}$ $1 \cdot 2 - 1 \cdot 4$

$2 - 4$

-2

• x: $\begin{vmatrix} 19 & 1 \\ -8 & 2 \end{vmatrix}$ $19 \cdot 2 - 1 \cdot -8$

$38 + 8$

46

• y: $\begin{vmatrix} 1 & 19 \\ 4 & -8 \end{vmatrix}$ $1 \cdot -8 - 19 \cdot 4$

$-8 - 76$

-84

$x = \frac{46}{-2}$ $y = \frac{-84}{-2}$

$(-23, 42)$

(16) $A_{5\times2}$ $B_{1\times5}$

$B \times A$
1×5 5×2

product would be 1×2

* find the Determinant!

(17)
$$\begin{bmatrix} -5 & 5 & -3 \\ 0 & 0 & 0 \\ 4 & -6 & -6 \end{bmatrix} \begin{matrix} -5 & 5 \\ 0 & 0 \\ 4 & -6 \end{matrix}$$

$(-5\cdot0\cdot-6)+(5\cdot0\cdot4)+(-3\cdot0\cdot-6)-(5\cdot0\cdot-6)-(-5\cdot0\cdot-6)-(-3\cdot0\cdot4)$

0

* there will NOT be an inverse!

(18)
$$\begin{bmatrix} -4 & 0 & 0 \\ 5 & 0 & 1 \\ 0 & 4 & -3 \end{bmatrix} \begin{matrix} -4 & 0 \\ 5 & 0 \\ 0 & 4 \end{matrix}$$

$(-4\cdot0\cdot-3)+(0\cdot1\cdot0)+(0\cdot5\cdot4)-(0\cdot5\cdot-3)-(-4\cdot1\cdot4)-(0\cdot0\cdot0)$

$0+0+0-0+16-0$

16

* there WILL be an inverse!

(19)
$$\begin{cases} x+y+z=3 \\ 3y+2z=3 \\ x-2z=2 \end{cases}$$

• denominator: $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & -2 \end{vmatrix}$

-7

• x: $\begin{vmatrix} 3 & 1 & 1 \\ 3 & 1 & 2 \\ 2 & 0 & -2 \end{vmatrix}$

2

• y: $\begin{vmatrix} 1 & 3 & 1 \\ 0 & 3 & 2 \\ 1 & 2 & -2 \end{vmatrix}$

-7

• z: $\begin{vmatrix} 1 & 1 & 3 \\ 0 & 3 & 3 \\ 1 & 0 & 2 \end{vmatrix}$

0

$\left(\frac{-2}{7}, 1, 0\right)$

(19) $\begin{cases} x+y+z=3 \\ 3y+2z=3 \\ x-2z=2 \end{cases}$ *only wants to solve for z! ☺

• denominator: $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 3 & 2 \\ 1 & 0 & -2 \end{vmatrix} = -7$ • z: $\begin{vmatrix} 1 & 1 & 3 \\ 0 & 3 & 3 \\ 1 & 0 & 2 \end{vmatrix} = 0$

$z = 0/-7$

$\boxed{z = 0}$

(20) $\begin{cases} x+3y-4z=9 \\ y+2z=8 \\ z=1 \end{cases}$

matrix equation:

$$\begin{bmatrix} 1 & 3 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 1 \end{bmatrix}$$

calc set-up:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 3 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 8 \\ 1 \end{bmatrix}$$

$\boxed{(-5, 6, 1)}$

(21) $\begin{cases} x+2y-6z=23 \\ x+3y+z=4 \\ 2x+5y-4z=24 \end{cases}$

matrix equation:

$$\begin{bmatrix} 1 & 2 & -6 \\ 1 & 3 & 1 \\ 2 & 5 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23 \\ 4 \\ 24 \end{bmatrix}$$

calc set-up:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & -6 \\ 1 & 3 & 1 \\ 2 & 5 & -4 \end{bmatrix}^{-1} \begin{bmatrix} 23 \\ 4 \\ 24 \end{bmatrix}$$

$\boxed{(1, 2, -3)}$

(22) c = price of coffee d = cost of doughnuts

$$\begin{cases} 10c + 5d = 16.50 \\ 5c + 10d = 14.25 \end{cases}$$

matrix eqn:

$$\begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 16.50 \\ 14.25 \end{bmatrix}$$

calc set-up:

$$\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 16.50 \\ 14.25 \end{bmatrix}$$

coffee = $1.25

(23) p = # of pennies n = # of nickels d = # of dimes

$$\begin{cases} p + n + d = 17 \\ .01p + .05n + .10d = 0.47 \\ 4n = p \ \star \rightarrow -p + 4n = 0 \end{cases}$$

matrix eqn:

$$\begin{bmatrix} 1 & 1 & 1 \\ .01 & .05 & .10 \\ -1 & 4 & 0 \end{bmatrix} \begin{bmatrix} p \\ n \\ d \end{bmatrix} = \begin{bmatrix} 17 \\ 0.47 \\ 0 \end{bmatrix}$$

calc set-up:

$$\begin{bmatrix} p \\ n \\ d \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ .01 & .05 & .10 \\ -1 & 4 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 17 \\ .47 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} p \\ n \\ d \end{bmatrix} = \begin{bmatrix} 12 \\ 3 \\ 2 \end{bmatrix}$$

12 pennies, 3 nickels, 2 dimes

(24) m = # of lbs meat p = # of lbs potatoes c = # of lbs carrots

$$\begin{cases} m + p + c = 13.5 \\ 6m + 3p + c = 48.50 \\ 2p = c \longrightarrow 2p - c = 0 \end{cases}$$

matrix equ:

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} m \\ p \\ c \end{bmatrix} = \begin{bmatrix} 13.5 \\ 48.5 \\ 0 \end{bmatrix}$$

calc set-up:

$$\begin{bmatrix} m \\ p \\ c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 3 & 1 \\ 0 & 2 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 13.5 \\ 48.5 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} m \\ p \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 2.5 \\ 5 \end{bmatrix}$$

6 lbs of meat, 2½ lbs of potatoes, 5 lbs of carrots

(25) a = # of lbs of apples b = # of lbs of bananas
r = # of lbs of oranges

$$\begin{cases} a + r + b = 60 \\ 2a + 5r + 3b = 180 \\ b + 10 = a \longrightarrow -a + b = -10 \end{cases}$$

matrix equ:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 3 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ r \\ b \end{bmatrix} = \begin{bmatrix} 60 \\ 180 \\ -10 \end{bmatrix}$$

calc set-up:

$$\begin{bmatrix} a \\ r \\ b \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 3 \\ -1 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 60 \\ 180 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} a \\ r \\ b \end{bmatrix} = \begin{bmatrix} 28 \\ 14 \\ 18 \end{bmatrix}$$

28 lbs of apples, 14 lbs of oranges, 18 lbs of bananas