

HW 2 - Permutations HW

① 1 2 3 4 5 6 7 8

(a) no repetition

$$\underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} = \boxed{1680}$$

(b) repetition is allowed

$$\underline{8} \cdot \underline{8} \cdot \underline{8} \cdot \underline{8} = \boxed{4096}$$

(c) no repetition, must be greater than 5000

must be 5, 6, 7, or 8 → $\underline{4} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} = \boxed{840}$
↑ can't be 1st #

(d) repetition is allowed, must be even

$$\underline{8} \cdot \underline{8} \cdot \underline{8} \cdot \underline{4} \leftarrow \text{must be even } 2, 4, 6, \text{ or } 8 = \boxed{2048}$$

(e) no repetition, must be less than 3000

must be 1 or 2 → $\underline{2} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} = \boxed{420}$

② $\underline{8} \cdot \underline{7} \cdot \underline{6}$ or $P(8, 3) = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{5!}} = \boxed{336}$

③ $\underline{100} \cdot \underline{99} \cdot \underline{98}$ or $P(100, 3) = \frac{100!}{97!} = \frac{100 \cdot 99 \cdot 98 \cdot \cancel{97!}}{\cancel{97!}} = \boxed{970,200}$

④ (a) $\underline{12} \cdot \underline{11} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8}$ or $P(12, 5) = \frac{12!}{7!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} = \boxed{95040}$

(b) $\underline{12} \cdot \underline{11} \cdot \text{Grandma Ling} \cdot \underline{10} \cdot \underline{9}$ or $P(12, 4) = \frac{12!}{8!} = \boxed{11,880}$

⑤ (a) $\underline{20} \cdot \underline{19} \cdot \underline{18}$
 or $P(20, 3) = \frac{20!}{17!} = \boxed{6840}$

(b) $\underline{20} \cdot \underline{20} \cdot \underline{20} = \boxed{8000}$

~~11A 840~~

⑥ MATCHING 8 total letters

a) $8! = 40320$

b) $\frac{8!}{7!} = 5040$

c) $\frac{8!}{1! \cdot 4! \cdot 3! \cdot 5! \cdot 2! \cdot 7!}$

$4! = 24$

d) $\frac{8!}{6!} = 720$

$6! = 720$

⑦ a) MAXIMUM

7 total M → 3

$\frac{7!}{3!} = 840$

b) CANADA

6 total A → 3

$\frac{6!}{3!} = 120$

c) BOOKKEEPER

10 total O → 2 E → 3

$\frac{10!}{2! \cdot 2! \cdot 3!} = 151,200$

d) MATHEMATICS

11 total M → 2

A → 2 T → 2

$\frac{11!}{2! \cdot 2! \cdot 2!} = 4,989,600$

⑧ 44666 5 total #s

$\frac{5!}{2! \cdot 3!} = 10$

⑨ 2 vanilla 3 chocolate 4 strawberry 1 pistachio

$\frac{10!}{2! \cdot 3! \cdot 4!} = 12,600$

- (10) 4 different books, 3 copies of each \rightarrow like copies must stay together \star

$$\frac{4! \cdot 3! \cdot 3! \cdot 3! \cdot 3!}{3! \cdot 3! \cdot 3! \cdot 3!} = 4! = 24$$

- (11) BASKETBALL 10 total letters B \rightarrow 2 A \rightarrow 2 L \rightarrow 2

of permutations: $\frac{10!}{2!2!2!} = 453,600$

(a) begin with K: $\frac{9!}{2!2!2!}$

45,360

(b) begin with B: $\frac{9!}{2!2!}$

90,720

- (c) two L's together \rightarrow treat them as 1 letter

$$\frac{9!}{2!2!} = 90,720$$

(12) $\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 720$

$6!$
 $P(6,6) = 6! = 720$

(13) $\underline{10} \cdot \underline{5} \cdot \underline{4} \cdot \underline{4} \cdot \underline{3} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1}$

\uparrow
 either

Boy or girl could be first in line!

$$10 \cdot 5! \cdot 4!$$

28,800