HW – APPLICATIONS DAY 2

NAME key F 17

1. If you start with one gram of a certain isotope, you have 0.9 grams after a week. → 7 days

a) What is the half life of this isotope?

$0.9 = 1(\frac{1}{2})^{7/x}$

$0.9 = (\frac{1}{2})^{7/x}$

$\log_{1/2} 0.9 = 7/x$

$.152 = 7/x$

$x(.152) = 7$

$x = 7/.152$

$x = 46.05$

$x \sim 46$ days

b) How much is left after a year? 365 days

$y = 1(\frac{1}{2})^{365/46.05}$

.00411 grams

c) How long will it be until there is $\frac{1}{10}$ gram left?

$\frac{1}{10} = (\frac{1}{2})^{x/46.05}$

$\log_{\frac{1}{2}} \frac{1}{10} = x/46.05$

$3.32193 = \frac{x}{46.05}$

$x \sim 153$ days

2. The bacteria *Escherichi coli* are commonly found in the bladder of human beings. Suppose that 3000 of the bacteria are present at time t = 0. If the amount of bacteria doubles every 20 minutes, find the following:

a) Find a formula representing the number of bacteria after t minutes.

$y = 3000(2)^{t/20}$

b) How many bacteria will be present after 10 minutes? 20 minutes? 2 hours?

10 mins: $3000(2)^{10/20} = 4,243$

30 mins: $3000(2)^{30/20} = 8,485$

2 hrs: $3000(2)^{120/20} = 192,000$

c) When will 1,000,000 bacteria be present?

$1,000,000 = 3,000(2)^{t/20}$

$\frac{1000}{3} = 2^{t/20}$

$\log_2 \frac{1000}{3} = t/20$

$8.38082 = t/20$

$t = 167.6$ mins or ~2 hrs & 48 mins

3. Suppose you start with 100 grams of a radioactive substance which has a half-life of 25 days.

a) How many grams are left after 40 days?

$y = 100(\frac{1}{2})^{40/25}$

32.99 grams

b) How long will it take until there are 5 grams left?

$5 = 100(\frac{1}{2})^{x/20}$

$.05 = (\frac{1}{2})^{x/20}$

$\log_{1/2} .05 = x/20$

$x \sim 108$ days

4. The annual interest rate on a $100,000 loan is 10%. Assuming no principle is paid, how long does it take to build up the interest charge to over $60,000 if it is compounded annually? Monthly? Continuously? *(The interest charge is the amount you pay for borrowing the loan money in the first place)*

ANNUALLY:

$160,000 = 100,000(1 + \frac{.10}{1})^{1t}$

$1.6 = 1.1^t$

$\log_{1.1} 1.6 = t$

$t = 4.93$ yrs

MONTHLY

$160,000 = 100,000(1 + \frac{.10}{12})^{12t}$

$1.6 = 1.008333^{12t}$

$\log_{1.008333} 1.6 = 12t$

$t = 4.72$ yrs

CONTINUOUSLY

$160,000 = 100,000e^{.10t}$

$1.6 = e^{.10t}$

$\ln 1.6 = .10t$

$t = 4.7$ yrs

5. Which would give you the most money if you invest $5000 for 2 years:

a) 10% compounded annually

$A = 5000\left(1 + \frac{.10}{1}\right)^{1(2)}$

$A = 5000(1.10)^2$

$6050

b) $9\frac{7}{8}$% compounded monthly

$A = 5000\left(1 + \frac{.09875}{12}\right)^{12(2)}$

$A = 5000(1.008229)^{24}$

$6086.84

c) $9\frac{3}{4}$ compounded continuously

$A = 5000e^{(.0975)(2)}$

$A = 5000e^{.195}$

$6076.55

6. This year an estimated 4,324,000 people in this country are illiterate. With new incentives and funding, the country is hoping to cut that number by 11% *every 3 years*. How many people do you predict will be illiterate in the year . . .

a) 2037? 20 years from now

$y = 4324000(.89)^{20/3}$

$\sim 1,988,317$

b) 2096? 79 years from now

$y = 4324000(.89)^{79/3}$

$\sim 200,981$

7. A cup of coffee contains approximately 96 mg of caffeine. When you drink the coffee, the caffeine is absorbed into the bloodstream and is eventually metabolized by the body. Every 5 hours the amount of caffeine present in the body is reduced by one-half. How many hours does it take for the amount of caffeine to be reduced to 12 mg?

$12 = 96(1/2)^{x/5}$

$.125 = (1/2)^{x/5}$

$\log_{\frac{1}{2}} .125 = x/5$

$x = 15$ hours

8. Sodium-24 has a half-life of 15 hours. How much sodium-24 will you have after 60 hours if your original sample is 64 mg?

$y = 64(1/2)^{60/15}$

4 mg

9. Find the amount in an account after 15 years if $5000 was initially invested and the account earns 8% annual interest compounded quarterly.

$A = 5000\left(1 + \frac{.08}{4}\right)^{4(15)}$

$16,405.15

10. Find the length of time needed to earn $124.49 if Megan invests $957.62 at 6.5% interest, compounded continuously.

this is her INTEREST... "A" is total $ accumulated

$1082.11 = 957.62e^{.065t}$

$1.129999 = e^{.065t}$

$\ln 1.129999 = .065t$

$t = 1.88$ years