Using Theorems to Find Zeros Day one: 1-20

① $x^3 - x^2 + 3x - 3 = 0$

$p = 3$: 1, 3 $\quad q = 1$: 1

$\pm 1, 3$

② $x^3 - 10x^2 + 17x - 8 = 0$

$p = 8$: 1, 2, 4, 8 $\quad q = 1$: 1

$\pm 1, 2, 4, 8$

③ $x^4 + x^3 - 10x^2 - 4x + 24 = 0$

$p = 24$: 1, 2, 3, 4, 6, 8, 12, 24 $\quad q = 1$: 1

$\pm 1, 2, 3, 4, 6, 8, 12, 24$

④ $2x^3 - 7x^2 + 7x - 2 = 0$

$p = 2$: 1, 2 $\quad q = 2$: 1, 2

$\pm 1, \frac{1}{2}, 2$

⑤ $5x^3 - 9x^2 - 17x - 3 = 0$

$p = 3$: 1, 3 $\quad q = 5$: 1, 5

$\pm 1, \frac{1}{5}, 3, \frac{3}{5}$

⑥ $3x^4 + 11x^3 - x^2 + 11x - 4 = 0$

$p = 4$: 1, 2, 4 $\quad q = 3$: 1, 3

$\pm 1, \frac{1}{3}, 2, \frac{2}{3}, 4, \frac{4}{3}$

⑦ $3x^4 - 25x^3 + 11x^2 - 25x + 8 = 0$

$p = 8$: 1, 2, 4, 8 $\quad q = 3$: 1, 3

$\pm 1, \frac{1}{3}, 2, \frac{2}{3}, 4, \frac{4}{3}, 8, \frac{8}{3}$

⑧ $3x^4 + 29x^3 - 7x^2 + 29x - 10$

$p = 10$: 1, 2, 5, 10 $\quad q = 3$: 1, 3

$\pm 1, \frac{1}{3}, 2, \frac{2}{3}, 5, \frac{5}{3}, 10, \frac{10}{3}$

(9) $f(x) = x^3 - 2x^2 + x - 3$

P → N → P → N

3 changes

3 or 1 + ℝ

$f(-x) = (-x)^3 - 2(-x)^2 + (-x) - 3$

$f(-x) = -x^3 - 2x^2 - x - 3$

N

0 changes

0 − ℝ

(10) $f(x) = x^3 - 4x^2 + x + 2$

P → N → P

2 changes

2 or 0 + ℝ

$f(-x) = (-x)^3 - 4(-x)^2 + (-x) + 2$

$f(-x) = -x^3 - 4x^2 - x + 2$

N → P

1 change

1 − ℝ

(11) $g(x) = 4x^3 + 5x^2 + 2x - 6$

P → N

1 change

1 + ℝ

$g(-x) = 4(-x)^3 + 5(-x)^2 + 2(-x) - 6$

$g(-x) = -4x^3 + 5x^2 - 2x - 6$

N → P → N

2 changes

2 or 0 − ℝ

(12) $h(x) = 2x^3 + 3x^2 - 4x - 1$

P → N

1 change

1 + ℝ

$h(-x) = 2(-x)^3 + 3(-x)^2 - 4(-x) - 1$

$h(-x) = -2x^3 + 3x^2 + 4x - 1$

N → P → N

2 changes

2 or 0 − ℝ

13 $h(x) = 2x^3 + 5x^2 - 4x - 5$

P ⟶ N

1 change

1 + IR

$h(-x) = 2(-x)^3 + 5(-x)^2 - 4(-x) - 5$

$h(-x) = -2x^3 + 5x^2 + 4x - 5$

N → P ⟶ N

2 changes

2 or 0 - IR

14 $f(x) = 3x^3 - 2x^2 + 6x - 1$

P → N → P → N

3 changes

3 or 1 + IR

$f(-x) = 3(-x)^3 - 2(-x)^2 + 6(-x) - 1$

$f(-x) = -3x^3 - 2x^2 - 6x - 1$

N

0 changes

0 - IR

15 $h(x) = 3x^3 + 2$

P

0 changes

0 + IR

$h(-x) = 3(-x)^3 + 2$

$h(-x) = -3x^3 + 2$

N → P

1 change

1 - IR

16 $g(x) = x^3 - 2x^2 - 3x$

P → N

1 change

1 + IR

$g(-x) = (-x)^3 - 2(-x)^2 - 3(-x)$

$g(-x) = -x^3 - 2x^2 + 3x$

N ⟶ P

1 change

1 - IR

(11) $g(x) = 2x^3 + 3x^2 - 8x + 3$

P → N → P

2 changes

2 or 0 +R

$g(-x) = 2(-x)^3 + 3(-x)^2 - 8(-x) + 3$

$g(-x) = -2x^3 + 3x^2 + 8x + 3$

N → P

1 change

1 −R

+R	2	0
−R	1	1
imag/irr	0	2

(18) $g(x) = 6x^3 - 11x^2 - 24x + 9$

P → N → P

2 changes

2 or 0 +R

$g(-x) = 6(-x)^3 - 11(-x)^2 - 24(-x) + 9$

$g(-x) = -6x^3 - 11x^2 + 24x + 9$

N → P

1 change

1 −R

+R	2	0
−R	1	1
imag/irr	0	2

(19) $h(x) = 2x^3 - 5x^2 + 2x - 4$

P → N → P → N

3 changes

3 or 1 +ℝ

$h(-x) = 2(-x)^3 - 5(-x)^2 + 2(-x) - 4$

$h(-x) = -2x^3 - 5x^2 - 2x - 4$

N

0 changes

0 −ℝ

+ℝ	3	1
−ℝ	0	0
imag/irr	0	2

(20) $f(x) = 5x^3 + 6x^2 - x - 1$

P → N

1 change

1 +ℝ

$f(-x) = 5(-x)^3 + 6(-x)^2 - (-x) - 1$

$f(-x) = -5x^3 + 6x^2 + x - 1$

N → P → N

2 changes

2 or 0 −ℝ

+ℝ	1	1
−ℝ	2	0
imag/irr	0	2