Honors Math 3 - Finding Roots of Polynomials

① $x^3 - x^2 + 3x - 3 = 0$ $\boxed{\pm 1,\ 3}$

$p = 3$, $q = 1$

1, 3 1

② $x^3 - 10x^2 + 17x - 8 = 0$ $\boxed{\pm 1, 2, 4, 8}$

$p = 8$ $q = 1$

1, 2, 4, 8 1

③ $x^4 + x^3 - 10x^2 - 4x + 24 = 0$ $\boxed{\pm 1, 2, 3, 4, 6, 12, 24}$

$p = 24$ $q = 1$

1, 2, 3, 4, 6,
8, 12, 24 1

④ $2x^3 - 7x^2 + 7x - 2 = 0$ $\boxed{\pm 1, \tfrac{1}{2}, 2}$

$p = 2$ $q = 2$

1, 2 1, 2

⑤ $5x^3 - 9x^2 - 17x - 3 = 0$ $\boxed{\pm 1, \tfrac{1}{5}, 3, \tfrac{3}{5}}$

$p = 3$ $q = 5$

1, 3 1, 5

⑥ $3x^4 + 11x^3 - x^2 + 11x - 4 = 0$ $\boxed{\pm 1, \tfrac{1}{3}, 2, \tfrac{2}{3}, 4, \tfrac{4}{3}}$

$p = 4$ $q = 3$

1, 2, 4 1, 3

⑦ $3x^4 - 25x^3 + 11x^2 - 25x + 8 = 0$ $\boxed{\pm 1, \tfrac{1}{3}, 2, \tfrac{2}{3}, 4, \tfrac{4}{3}, 8, \tfrac{8}{3}}$

$p = 8$ $q = 3$

1, 2, 4, 8 1, 3

(8) $3x^4+29x^3-7x^2+29x-10=0$

$p=10$ $q=3$

1, 2, 5, 10 1, 3

$\boxed{\pm 1, \frac{1}{3}, 2, \frac{2}{3}, 5, \frac{5}{3}, 10, \frac{10}{3}}$

(9) $f(x)=x^3-2x^2+x-3$

$P \to N \to P \to N$: 3 changes

$f(-x)=(-x)^3-2(-x)^2+(-x)-3$

$f(-x)=-x^3-2x^2-x-3$: 0 changes

N

$\boxed{\begin{array}{ll} 3 \text{ or } 1 & +\mathbb{R} \\ 0 & -\mathbb{R} \end{array}}$

(10) $f(x)=x^3-4x^2+x+2$

$P \to N \to P$: 2 changes

$f(-x)=(-x)^3-4(-x)^2+(-x)+2$

$f(-x)=-x^3-4x^2-x+2$

$N \longrightarrow P$: 1 change

$\boxed{\begin{array}{ll} 2 \text{ or } 0 & +\mathbb{R} \\ 1 & -\mathbb{R} \end{array}}$

(11) $g(x)=4x^3+5x^2+2x-6$

$P \longrightarrow N$: 1 change

$g(-x)=4(-x)^3+5(-x)^2+2(-x)-6$

$g(-x)=-4x^3+5x^2-2x-6$

$N \to P \to N$: 2 changes

$\boxed{\begin{array}{ll} 1 & +\mathbb{R} \\ 2 \text{ or } 0 & -\mathbb{R} \end{array}}$

(12) $h(x)=2x^3+3x^2-4x-1$

$P \longrightarrow N$: 1 change

$h(-x)=2(-x)^3+3(-x)^2-4(-x)-1$

$h(-x)=-2x^3+3x^2+4x-1$

$N \to P \longrightarrow N$: 2 changes

$\boxed{\begin{array}{ll} 1 & +\mathbb{R} \\ 2 \text{ or } 0 & -\mathbb{R} \end{array}}$

(13) $h(x) = 2x^3 + 5x^2 - 4x - 5$

$P \longrightarrow N$: 1 change

$h(-x) = 2(-x)^3 + 5(-x)^2 - 4(-x) - 5$

$h(-x) = -2x^3 + 5x^2 + 4x - 5$

$N \rightarrow P \longrightarrow N$: 2 changes

1	+ R
2 or 0	− R

(14) $f(x) = 3x^3 - 2x^2 + 6x - 1$

$P \rightarrow N \rightarrow P \rightarrow N$: 3 changes

$f(-x) = 3(-x)^3 - 2(-x)^2 + 6(-x) - 1$

$f(-x) = -3x^3 - 2x^2 - 6x - 1$

0 changes

3 or 1	+ R
0	− R

(15) $h(x) = 3x^3 + 2$

0 changes

$h(-x) = 3(-x)^3 + 2$

$h(-x) = -3x^3 + 2$

$N \rightarrow P$: 1 change

0	+ R
1	− R

(16) $g(x) = x^3 - 2x^2 - 3x$

$P \rightarrow N$: 1 change

$g(-x) = (-x)^3 - 2(-x)^2 - 3(-x)$

$g(-x) = -x^3 - 2x^2 + 3x$

$N \longrightarrow P$: 1 change

1	+ R
1	− R

(17) $g(x) = 2x^3 + 3x^2 - 8x + 3$

$P \longrightarrow N \rightarrow P$: 2 changes

$g(-x) = 2(-x)^3 + 3(-x)^2 - 8(-x) + 3$

$g(-x) = -2x^3 + 3x^2 + 8x + 3$

$N \rightarrow P$: 1 change

$$\boxed{\begin{array}{c} 2 \text{ or } 0 \quad +\mathbb{R} \\ 1 \quad -\mathbb{R} \end{array}}$$

(18) $g(x) = 6x^3 - 11x^2 - 24x + 9$

$P \rightarrow N \longrightarrow P$: 2 changes

$g(-x) = 6(-x)^3 - 11(-x)^2 - 24(-x) + 9$

$g(-x) = -6x^3 - 11x^2 + 24x + 9$

$N \longrightarrow P$: 1 change

$$\boxed{\begin{array}{c} 2 \text{ or } 0 \quad +\mathbb{R} \\ 1 \quad -\mathbb{R} \end{array}}$$

(19) $h(x) = 2x^3 - 5x^2 + 2x - 4$

$P \rightarrow N \rightarrow P \rightarrow N$: 3 changes

$h(-x) = 2(-x)^3 - 5(-x)^2 + 2(-x) - 4$

$h(-x) = -2x^3 - 5x^2 - 2x - 4$

0 changes

$$\boxed{\begin{array}{c} 3 \text{ or } 1 \quad +\mathbb{R} \\ 0 \quad -\mathbb{R} \end{array}}$$

(20) $f(x) = 5x^3 + 6x^2 - x - 1$

$P \longrightarrow N$: 1 change

$f(-x) = 5(-x)^3 + 6(-x)^2 - (-x) - 1$

$f(-x) = -5x^3 + 6x^2 + x - 1$

$N \rightarrow P \rightarrow N$: 2 changes

$$\boxed{\begin{array}{c} 1 \quad +\mathbb{R} \\ 2 \text{ or } 0 \quad -\mathbb{R} \end{array}}$$

(21) $x^3 - 4x^2 - 3x + 18 = 0$ should have 3 roots

$p = 18$ $q = 1$

1, 2, 3, 6, 9, 18

possible roots: $\pm 1, 2, 3, 6, 9, 18$

$f(x) = x^3 - 4x^2 - 3x + 18$

$P \to N \to P$: 2 changes 2 or 0 +IR

$f(-x) = (-x)^3 - 4(-x)^2 - 3(-x) + 18$

$f(-x) = -x^3 - 4x^2 + 3x + 18$

$N \to P$: 1 change 1 −IR

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -3 & 18 \\ + & \downarrow & -2 & 12 & -18 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$x^2 - 6x + 9 = 0$

$(x-3)(x-3) = 0$

$x = 3$ (multiplicity 2)

double root

ROOTS:

$x = -2, 3$ (multiplicity 2)

22 $x^3 - x^2 - 8x + 12 = 0$ 3 roots

$p = 12$ $q = 1$ $\pm 1, 2, 3, 4, 6, 12$

1, 2, 3, 4,
6, 12

$f(x) = x^3 - x^2 - 8x + 12 = 0$

$P \rightarrow N \longrightarrow P$: 2 changes 2 or 0 + R

$f(-x) = (-x)^3 - (-x)^2 - 8(-x) + 12$ 1 − R

$f(-x) = -x^3 - x^2 + 8x + 12$

$N \longrightarrow P$: 1 change

$$
\begin{array}{r|rrrr}
2 & 1 & -1 & -8 & 12 \\
+ & \downarrow & 2 & 2 & -12 \\
\hline
 & 1 & 1 & -6 & 0
\end{array}
$$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ $x = 2$

double root

ROOTS:

$x = 2$ (multiplicity 2), -3

(23) $x^3-5x^2+4x-20$ 3 roots

$\pm 1, 2, 4, 5, 10, 20$

$p=20$ $q=1$

1, 2, 4, 5, 10, 20

$f(x)=x^3-5x^2+4x-20$

$P \to N \to P \to N$: 3 changes

$f(-x)=(-x)^3-5(-x)^2+4(-x)-20$

$f(-x)=-x^3-5x^2-4x-20$

0 changes

3 or 1 $+\mathbb{R}$

0 $-\mathbb{R}$

$$\begin{array}{r|rrrr} 5 & 1 & -5 & 4 & -20 \\ + & \downarrow & 5 & 0 & 20 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2+4=0$

$x^2=-4$

$x=\pm\sqrt{-4}$ $x=\pm 2i$

$$\boxed{\text{ROOTS: } 5, \pm 2i}$$

(24) $x^3+9x^2+23x+15=0$ 3 roots

$p=15$ $q=1$ $\pm 1, 3, 5, 15$

1, 3, 5, 15

$f(x)=x^3+9x^2+23x+15$

0 changes

$f(-x)=(-x)^3+9(-x)^2+23(-x)+15$

$f(-x)=-x^3+9x^2-23x+15$

$N \to P \to N \to P$ 3 changes

0 $+\mathbb{R}$

3 or 1 $-\mathbb{R}$

$$\begin{array}{r|rrrr} -1 & 1 & 9 & 23 & 15 \\ + & \downarrow & -1 & -8 & -15 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$

$x^2+8x+15=0$

$(x+3)(x+5)=0$

$x=-3$ $x=-5$

$$\boxed{\text{ROOTS: } x=-1, -3, -5}$$

(25) $x^3+x^2-2x-2=0$ 3 roots

$p=2$ $q=1$ $\pm 1, 2$

$f(x)=x^3+x^2-2x-2$

P → N : 1 change

$f(-x)=(-x)^3+(-x)^2-2(-x)-2$

$f(-x)=-x^3+x^2+2x-2$

N → P → N : 2 changes

1 +IR

2 or 0 -IR

-1	1	1	-2	-2
+	↓	-1	0	2
	1	0	-2	0

$x^2-2=0$

$x^2=2$

$x=\pm\sqrt{2}$

ROOTS:

$x=-1, \pm\sqrt{2}$

(26) $x^3-x^2-14x+24=0$ 3 roots

$p=24$ $q=1$ $\pm 1, 2, 3, 4, 6, 12, 24$

1, 2, 3, 4, 6, 12, 24

$f(x)=x^3-x^2-14x+24$

P → N → P : 2 changes

$f(-x)=(-x)^3-(-x)^2-14(-x)+24$

$f(-x)=-x^3-x^2+14x+24$

N → P : 1 change

2 or 0 +IR

1 -IR

2	1	-1	-14	24
+	↓	2	2	-24
	1	1	-12	0

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4$ $x=3$

ROOTS:

$x=2, 3, -4$

(27) $x^3-4x^2-7x+10=0$ 3 roots

$p=10$ $q=1$ $\pm 1, 2, 5, 10$

1, 2, 5, 10

$f(x)=x^3-4x^2-7x+10$

P → N → P : 2 changes

$f(-x)=(-x)^3-4(-x)^2-7(-x)+10$

$f(-x)=-x^3-4x^2+7x+10$

N → P 1 change

2 or 0 +IR

1 -IR

1	1	-4	-7	10
+	↓	1	-3	-10
	1	-3	-10	0

$x^2-3x-10=0$

$(x-5)(x+2)=0$

$x=5$ $x=-2$

ROOTS:

$x=1, 5, -2$

28) $f(x) = x^3 - x^2 - 2x + 8 = 0$ 3 roots

$p = 8$ $q = 1$ $\pm 1, 2, 4, 8$

$f(x) = x^3 - x^2 - 2x + 8$

P → N → P : 2 changes

$f(-x) = (-x)^3 - (-x)^2 - 2(-x) + 8$

$f(-x) = -x^3 - x^2 + 2x + 8$

N → P : 1 change

2 or 0 $+\mathbb{R}$

1 $-\mathbb{R}$

$$\begin{array}{r|rrrr} -2 & 1 & -1 & -2 & 8 \\ + & \downarrow & -2 & 6 & -8 \\ \hline & 1 & -3 & 4 & 0 \end{array}$$

$x^2 - 3x + 4 = 0$

$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(4)}}{2(1)}$

$x = \dfrac{3 \pm \sqrt{-7}}{2}$

$$\boxed{\text{ROOTS: } x = -2, \ \frac{3 \pm i\sqrt{7}}{2}}$$

29) $x^3 - 19x - 30 = 0$ 3 roots

$p = 30$ $q = 1$ $\pm 1, 2, 3, 5, 6, 10, 30$

1, 2, 3, 5, 6, 10, 30

$f(x) = x^3 - 19x - 30$

P → N : 1 change

$f(-x) = (-x)^3 - 19(-x) - 30$

$f(-x) = -x^3 + 19x - 30$

N → P → N : 2 changes

1 $+\mathbb{R}$

2 or 0 $-\mathbb{R}$

$$\begin{array}{r|rrrr} 5 & 1 & 0 & -19 & -30 \\ + & \downarrow & 5 & 25 & 30 \\ \hline & 1 & 5 & 6 & 0 \end{array}$$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x = -3$ $x = -2$

$$\boxed{\text{ROOTS: } x = 5, -3, -2}$$

(30) ~~f(x)=~~ $2x^3+5x^2+6x+2=0$ 3 roots

$p=2$ $q=2$ $\pm 1, \frac{1}{2}, 2$

1, 2 1, 2

$f(-x)=2(-x)^3+5(-x)^2+6(-x)+2$

$f(x)=2x^3+5x^2+6x+2$

$f(-x)=-2x^3+5x^2-6x+2$

0 changes

N → P → N → P 3 changes

0 +ℝ

3 or 1 −ℝ

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 5 & 6 & 2 \\ + & \downarrow & -1 & -2 & -2 \\ \hline & 2 & 4 & 4 & 0 \end{array}$$

*can ÷ by 2 $2x^2+4x+4=0$

$x^2+2x+2=0$

$a=1$ $b=2$ $c=2$

$x=\frac{-2\pm\sqrt{(2)^2-4(1)(2)}}{2(1)}$

$x=\frac{-2\pm\sqrt{-4}}{2}$

$x=\frac{-2\pm 2i}{2}$ *can simplify

ROOTS:

$x=\frac{-1}{2}, -1\pm i$

(31) $3x^3-5x^2+14x-8=0$ 3 roots

$p=8$ $q=3$ $\pm 1, \frac{1}{3}, 2, \frac{2}{3}, 4, \frac{4}{3}, 8, \frac{8}{3}$

1, 2, 4, 8 1, 3

$f(-x)=3(-x)^3-5(-x)^2+14(-x)-8$

$f(x)=3x^3-5x^2+14x-8$

$f(-x)=-3x^3-5x^2-14x-8$

P → N → P → N 3 changes

0 changes

3 or 1 +ℝ

0 −ℝ

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -5 & 14 & -8 \\ + & \downarrow & 2 & -2 & 8 \\ \hline & 3 & -3 & 12 & 0 \end{array}$$

$x=\frac{1\pm\sqrt{(-1)^2-4(1)(4)}}{2(1)}$

$3x^2-3x+12=0$

$x=\frac{1\pm\sqrt{-15}}{2}$

*can ÷ by 3

$x^2-x+4=0$

$a=1$ $b=-1$ $c=4$

$x=\frac{1\pm i\sqrt{15}}{2}$

ROOTS:

$x=\frac{2}{3}, \frac{1\pm i\sqrt{15}}{2}$

(32) $x^3 - 4x^2 - x + 4 = 0$ 3 roots

$p = 4$ $q = 1$ $\pm 1, 2, 4$

1, 2, 4

$f(x) = x^3 - 4x^2 - x + 4$

P → N → P : 2 changes

$f(-x) = (-x)^3 - 4(-x)^2 - (-x) + 4$

$f(-x) = -x^3 - 4x^2 + x + 4$

N → P : 1 change

2 or 0 +IR

1 −IR

$$\begin{array}{r|rrrr} -1 & 1 & -4 & -1 & 4 \\ + & \downarrow & -1 & 5 & -4 \\ \hline & 1 & -5 & 4 & 0 \end{array}$$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4$ $x = 1$

$$\boxed{\text{ROOTS: } x = -1, 4, 1}$$

(33) $x^3 - 2x + 4 = 0$ 3 roots

$p = 4$ $q = 1$ $\pm 1, 2, 4$

$f(x) = x^3 - 2x + 4$

P → N → P : 2 changes

$f(-x) = (-x)^3 - 2(-x) + 4$

$f(-x) = -x^3 + 2x + 4$

N → P : 1 change

2 or 0 +IR

1 − IR

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -2 & 4 \\ + & \downarrow & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$x^2 - 2x + 2 = 0$

$a = 1$ $b = -2$ $c = 2$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-4}}{2}$$

$$x = \frac{2 \pm 2i}{2}$$

☆ reduce!

$$\boxed{\text{ROOTS: } x = -2, 1 \pm i}$$

(34) $x^3 - 5x^2 - x + 5 = 0$ 3 roots

$p = 5 \quad q = 1 \qquad \pm 1, 5$

$f(x) = x^3 - 5x^2 - x + 5$

$P \rightarrow N \longrightarrow P$: 2 changes

$f(-x) = (-x)^3 - 5(-x)^2 - (-x) + 5$

$f(-x) = -x^3 - 5x^2 + x + 5$

$N \longrightarrow P$: 1 change

2 or 0 $+\mathbb{R}$

1 $-\mathbb{R}$

$$\begin{array}{r|rrrr} -1 & 1 & -5 & -1 & 5 \\ + & \downarrow & -1 & 6 & -5 \\ \hline & 1 & -6 & 5 & 0 \end{array}$$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 5 \quad x = 1$

$$\boxed{\text{ROOTS: } x = -1, 1, 5}$$

(35) $x^3 + 6x^2 + 12x + 8 = 0$

$p = 8 \qquad q = 1 \qquad \pm 1, 2, 4, 8$

$f(x) = x^3 + 6x^2 + 12x + 8$

P : 0 changes

$f(-x) = (-x)^3 + 6(-x)^2 + 12(-x) + 8$

$f(-x) = -x^3 + 6x^2 - 12x + 8$

$N \rightarrow P \rightarrow N \rightarrow P$: 3 changes

0 $+\mathbb{R}$

3 or 1 $-\mathbb{R}$

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 12 & 8 \\ + & \downarrow & -2 & -8 & -8 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$x^2 + 4x + 4 = 0$

$a = 1 \quad b = 4 \quad c = 4$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-4 \pm 0}{2}$$

$$x = -4/2$$

$$x = -2$$

$$\boxed{\text{ROOTS: } x = -2}$$

(36) $x^3-3x^2+x-3=0$ 3 roots

$p=3$ $q=1$ $\pm 1, 3$

$f(x)=x^3-3x^2+x-3$

$P \to N \to P \to N$: 3 changes

$f(-x)=(-x)^3-3(-x)^2+(-x)-3$

$f(-x)=-x^3-3x^2-x-3$

0 changes

3 or 1 +ℝ

0 −ℝ

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 1 & -3 \\ + & \downarrow & 3 & 0 & 3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}$

$x=\pm i$

$$\boxed{\text{ROOTS} \quad x = 3, \pm i}$$

(37) $x^3+5x^2+3x-34=0$ 3 roots

$p=34$ $q=1$ $\pm 1, 2, 17, 34$

$f(x)=x^3+5x^2+3x-34$

$P \longrightarrow N$: 1 change

$f(-x)=(-x)^3+5(-x)^2+3(-x)-34$

$f(-x)=-x^3+5x^2-3x-34$

$N \to P \to N$: 2 changes

1 +ℝ

2 or 0 −ℝ

$$\begin{array}{r|rrrr} 2 & 1 & 5 & 3 & -34 \\ + & \downarrow & 2 & 14 & 34 \\ \hline & 1 & 7 & 17 & 0 \end{array}$$

$x^2+7x+17=0$

$a=1$ $b=7$ $c=17$

$x=\dfrac{-7\pm\sqrt{(7)^2-4(1)(17)}}{2(1)}$

$x=\dfrac{-7\pm\sqrt{-19}}{2}$

$x=\dfrac{-7\pm i\sqrt{19}}{2}$

$$\boxed{\text{ROOTS: } \quad x = 2, \frac{-7\pm i\sqrt{19}}{2}}$$

(38) $x^3-8x+32=0$ 3 roots

$p=32$ $q=1$ $\pm 1, 2, 4, 8, 16, 32$

$f(x)=x^3-8x+32$

P → N → P : 2 changes

$f(-x)=(-x)^3-8(-x)+32$

$f(-x)=-x^3+8x+32$

N → P : 1 change

2 or 0 +ℝ

1 −ℝ

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -8 & 32 \\ + & \downarrow & -4 & 16 & -32 \\ \hline & 1 & -4 & 8 & 0 \end{array}$$

$x^2-4x+8=0$

$a=1$ $b=-4$ $c=8$

$x=\dfrac{4\pm\sqrt{(-4)^2-4(1)(8)}}{2(1)}$

$x=\dfrac{4\pm\sqrt{-16}}{2}$

$x=\dfrac{4\pm 4i}{2}$ ★reduce!

ROOTS:

$x=-4,$

$\dfrac{2\pm 2i}{1}$

(39) $x^4+4x^3-2x^2-12x+9=0$ 4 roots

$p=9$ $q=1$ $\pm 1, 3, 9$

$f(x)=x^4+4x^3-2x^2-12x+9$

P → N → P : 2 changes

$f(-x)=(-x)^4+4(-x)^3-2(-x)^2-12(-x)+9$

$f(-x)=x^4-4x^3-2x^2+12x+9$

P → N → P : 2 changes

2 or 0 +ℝ

2 or 0 −ℝ

$$\begin{array}{r|rrrrr} 1 & 1 & 4 & -2 & -12 & 9 \\ + & \downarrow & 1 & 5 & 3 & -9 \\ \hline & 1 & 5 & 3 & -9 & 0 \end{array}$$

$x^3+5x^2+3x-9=0$

$$\begin{array}{r|rrrr} -3 & 1 & 5 & 3 & -9 \\ + & \downarrow & -3 & -6 & 9 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3$ $x=1$

ROOTS:

$x=1, -3$

(40) $x^4+8x^3+24x^2+32x+16=0$ 4 roots

$p=1$ $q=16$ $\pm 1, 2, 4, 8, 16$

$f(x)=x^4+8x^3+24x^2+32x+16$

0 changes

$f(-x)=(-x)^4+8(-x)^3+24(-x)^2+32(-x)+16$

$f(-x)=x^4-8x^3+24x^2-32x+16$

$P \to N \to P \to N \to P$

4 changes

0 +R

4, 2, 0 −R

$$\begin{array}{r|rrrrr} -2 & 1 & 8 & 24 & 32 & 16 \\ + & \downarrow & -2 & -12 & -24 & -16 \\ \hline & 1 & 6 & 12 & 8 & 0 \end{array}$$

$x^3+6x^2+12x+8=0$

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 12 & 8 \\ + & \downarrow & -2 & -8 & -8 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$x^2+4x+4=0$

$(x+2)(x+2)=0$

$x=-2$ $x=-2$

$\boxed{\underline{\text{ROOTS:}} \quad x=-2}$

41) $x^4-4x^3+6x^2-4x+1=0$ 4 roots

$p=1$ $q=1$ ± 1

$f(x)=x^4-4x^3+6x^2-4x+1$

$P \to N \to P \to N \to P$: 4 changes

$f(-x)=(-x)^4-4(-x)^3+6(-x)^2-4(-x)+1$

$f(-x)=x^4+4x^3+6x^2+4x+1$

0 changes

4, 2, or 0 +R

0 −R

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 6 & -4 & 1 \\ + & \downarrow & 1 & -3 & 3 & -1 \\ \hline & 1 & -3 & 3 & -1 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1 \\ + & \downarrow & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$x^2-2x+1=0$

$(x-1)(x-1)=0$

$x=1$ $x=1$

$\boxed{\text{ROOTS: } x=1}$

42) $x^4-x^3-11x^2-5x+4=0$ 4 roots

$p=4$ $q=1$ $\pm 1, 2, 4$

$f(x)=x^4-x^3-11x^2-5x+4$

$P \to N \longrightarrow P$: 2 changes

$f(-x)=(-x)^4-(-x)^3-11(-x)^2-5(-x)+4$

$f(-x)=x^4+x^3-11x^2+5x+4$

$P \to N \to P$: 2 changes

2 or 0 +R

2 or 0 −R

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -11 & -5 & 4 \\ + & \downarrow & -1 & 2 & 9 & -4 \\ \hline & 1 & -2 & -9 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 1 & -2 & -9 & 4 \\ + & \downarrow & 4 & 8 & -4 \\ \hline & 1 & 2 & -1 & 0 \end{array}$$

$x^2+2x-1=0$

$x=\dfrac{-2\pm\sqrt{(2)^2-4(1)(-1)}}{2(1)}$

$x=\dfrac{-2\pm\sqrt{8}}{2}$

★ break $\sqrt{8}$ down

$4 \wedge 2$

$2\,2$

$x=\dfrac{-2\pm 2\sqrt{2}}{2}$ ★ reduce!

$\boxed{\text{ROOTS: } x=-1, 4, -1\pm\sqrt{2}}$

(43) $x^4+13x^2+36=0$ 4 roots

*can factor this!

$(x^2+9)(x^2+4)=0$

$x^2=-9 \quad x^2=-4$

$x=\pm\sqrt{-9} \quad x=\pm\sqrt{-4}$

$\boxed{\text{ROOTS: } x=\pm 3i, \pm 2i}$

(44) $x^4+x^2-20=0$ 4 roots

$p=20 \quad q=1 \quad \pm 1, 2, 4, 5, 10, 20$

$f(x)=x^4+x^2-20$

$P \to N$: 1 change

$f(-x)=(-x)^4+(-x)^2-20$

$f(-x)=x^4+x^2-20$

$P \to N$: 1 change

1 + IR

1 − IR

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 1 & 0 & -20 \\ & + \downarrow & 2 & 4 & 10 & 20 \\ \hline & 1 & 2 & 5 & 10 & 0 \end{array}$$

*Must fill in 0's for x^3 & x

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 5 & 10 \\ & + \downarrow & -2 & 0 & -10 \\ \hline & 1 & 0 & 5 & 0 \end{array}$$

$x^2+5=0$

$x^2=-5$

$x=\pm\sqrt{-5}$

$\boxed{\text{ROOTS: } x=2, -2, \pm i\sqrt{5}}$

(45) $4x^4+12x^3+19x^2-3x-5=0$ 4 roots

$p=5$ $q=4$ $\pm 1, \frac{1}{2}, \frac{1}{4}, 5, \frac{5}{2}, \frac{5}{4}$

1, 5 1, 2, 4

$f(x)=4x^4+12x^3+19x^2-3x-5$

P $\longrightarrow$ N : 1 change

1 + R

3 or 1 − R

$f(-x)=4(-x)^4+12(-x)^3+19(-x)^2-3(-x)-5$

$f(-x)=4x^4-12x^3+19x^2+3x-5$

P → N → P → N

3 changes

$$\begin{array}{r|rrrrr} \frac{1}{2} & 4 & 12 & 19 & -3 & -5 \\ + & \downarrow & 2 & 7 & 13 & 5 \\ \hline & 4 & 14 & 26 & 10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 14 & 26 & 10 \\ + & \downarrow & -2 & -6 & -10 \\ \hline & 4 & 12 & 20 & 0 \end{array}$$

$4x^2+12x+20=0$ ÷ by 4

$x^2+3x+5=0$

$x=\dfrac{-3\pm\sqrt{(3)^2-4(1)(5)}}{2(1)}$

$x=\dfrac{-3\pm\sqrt{-11}}{2}$

ROOTS:

$x=\frac{1}{2}, -\frac{1}{2}, \dfrac{-3\pm i\sqrt{11}}{2}$

(46) $2x^3+5x^2+6x+2=0$ 3 roots

$p=2$ $q=2$ $\pm 1, \frac{1}{2}, 2$

1, 2 1, 2

$f(x)=2x^3+5x^2+6x+2$

0 changes

0 + R

3 or 1 − R

$f(-x)=2(-x)^3+5(-x)^2+6(-x)+2$

$f(-x)=-2x^3+5x^2-6x+2$

N → P → N → P

3 changes

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 5 & 6 & 2 \\ + & \downarrow & -1 & -2 & -2 \\ \hline & 2 & 4 & 4 & 0 \end{array}$$

★ ÷ by 2 $2x^2+4x+4=0$

$x^2+2x+2=0$

$x=\dfrac{-2\pm\sqrt{(2)^2-4(1)(2)}}{2(1)}$

$x=\dfrac{-2\pm\sqrt{-4}}{2}$

$x=\dfrac{-2\pm 2i}{2}$ ★reduce!

ROOTS:

$x=-\frac{1}{2}, -1\pm i$