#M3 Equations of circles continued

① A (2, -5) B (6, -15) C (9, -8)

• ⊥ of AB

midpoint $\left(\frac{2+6}{2}, \frac{-5-15}{2}\right)$

mdpt (4, -10)

$m_{AB} = \frac{-5+(+15)}{2-6} = \frac{10}{-4} = -5/2$

$m_\perp = 2/5$

$y = mx + b$

$(-10) = (2/5)(4) + b$

$-10 = 8/5 + b$

$-58/5 = b$

$y = 2/5\,x - 58/5$

• ⊥ of BC

midpoint $\left(\frac{6+9}{2}, \frac{-15+-8}{2}\right)$

mdpt $(15/2, \frac{-23}{2})$

$m_{BC} = \frac{-15+(+8)}{6-9} = -7/-3 = 7/3$

$m_\perp = -3/7$

$y = mx + b$

$-\frac{23}{2} = (-3/7)(15/2) + b$

$-23/2 = -45/14 + b$

$-58/7 = b$

$y = -3/7\,x - 58/7$

• ⊥ of AC

midpoint $\left(\frac{2+9}{2}, \frac{-5-8}{2}\right)$

mdpt $(11/2, -13/2)$

$m_{AC} = \frac{-5+(+8)}{2-9} = 3/-7 = -3/7$

$m_\perp = 7/3$

$y = mx + b$

$-13/2 = (7/3)(11/2) + b$

$-13/2 = 77/6 + b$

$-58/3 = b$

$y = 7/3\,x - 58/3$

*now use any 2 of the above 3 eqns to solve *

⊥ AB, ⊥ BC:

$y = 2/5\,x - 58/5$

$y = -3/7\,x - 58/7$

$(2/5x - 58/5 = -3/7x - 58/7)\,35$

$14x - 406 = -15x - 290$

$29x = 116$

$x = 4$

$y = -10$

⊥ BC, ⊥ AC:

$y = -3/7\,x - 58/7$

$y = 7/3\,x - 58/3$

$(-3/7x - 58/7 = 7/3x - 58/3)\,21$

$-9x - 174 = 49x - 406$

$-58x = -232$

$x = 4$

$y = -10$

⊥ AB, ⊥ AC:

$y = 2/5\,x - 58/5$

$y = 7/3\,x - 58/3$

$(2/5x - 58/5 = 7/3\,x - 58/3)\,15$

$6x - 174 = 35x - 290$

$116 = 29x$

$x = 4$

$y = -10$

*now use distance formula w/ center and A, B, or C *

using point A: $r = \sqrt{(4-2)^2 + (-10--5)^2} = \sqrt{4+25} = \sqrt{29}$

using point B: $r = \sqrt{(4-6)^2 + (-10--15)^2} = \sqrt{4+25} = \sqrt{29}$

using point C: $r = \sqrt{(4-9)^2 + (-10--8)^2} = \sqrt{25+4} = \sqrt{29}$

$$(x-4)^2 + (y+10)^2 = 29$$

② A(-1, 11) B(1, 9) C(1, 13)

• ⊥ of AB

midpoint $\left(\frac{-1+1}{2}, \frac{11+9}{2}\right)$

mdpt (0, 10)

$m_{AB} = \frac{11-9}{-1-1} = \frac{2}{-2} = -1$

$m_{\perp} = 1$

$y = mx + b$

$10 = 1(0) + b$

$10 = b$

$y = x + 10$

• ⊥ of BC

midpoint $\left(\frac{1+1}{2}, \frac{9+13}{2}\right)$

mdpt (1, 11)

$m_{BC} = \frac{9-13}{1-1} = \frac{-4}{0}$

$m_{\perp} = 0$

$y = mx + b$

$11 = (0)(1) + b$

$11 = b$

$y = 11$

• ⊥ of AC

midpoint $\left(\frac{-1+1}{2}, \frac{11+13}{2}\right)$

mdpt (0, 12)

$m_{AC} = \frac{11-13}{-1-1} = \frac{-2}{-2} = 1$

$m_{\perp} = -1$

$y = mx + b$

$12 = (-1)(0) + b$

$12 = b$

$y = -x + 12$

$\perp AB \begin{cases} y = x + 10 \\ \perp BC \end{cases} \begin{cases} y = 11 \end{cases}$

$11 = x + 10$

$1 = x$

center (1, 11)

$\perp BC \begin{cases} y = 11 \\ \perp AC \end{cases} \begin{cases} y = -x + 12 \end{cases}$

$11 = -x + 12$

$-1 = -x$

center (1, 11)

$\perp AB \begin{cases} y = x + 10 \\ \perp AC \end{cases} \begin{cases} y = -x + 12 \end{cases}$

$x + 10 = -x + 12$

$2x = 2$

$x = 1$

center (1, 11)

now find r:

- using Point A: $r = \sqrt{(1+1)^2 + (11-11)^2} = \sqrt{4+0} = \sqrt{4} = 2$
- using Point B: $r = \sqrt{(1-1)^2 + (11-9)^2} = \sqrt{0+4} = \sqrt{4} = 2$
- using Point C: $r = \sqrt{(1-1)^2 + (11-13)^2} = \sqrt{0+4} = \sqrt{4} = 2$

$$(x-1)^2 + (y-11)^2 = 4$$

3) A(-4, 3) B(-12, 5) C(-9, 0)

• ⊥ of AB

midpoint $\left(\frac{-4-12}{2}, \frac{3+5}{2}\right)$

mdpt (-8, 4)

$m_{AB} = \frac{3-5}{-4-(-12)} = \frac{-2}{8} = \frac{-1}{4}$

$m_{\perp} = 4$

$y = mx + b$

$4 = (4)(-8) + b$

$4 = -32 + b$

$36 = b$

$y = 4x + 36$

• ⊥ of BC

midpoint $\left(\frac{-12-9}{2}, \frac{5+0}{2}\right)$

mdpt $\left(-21/2, 5/2\right)$

$m_{BC} = \frac{5-0}{-12-(-9)} = \frac{5}{-3}$

$m_{\perp} = 3/5$

$y = mx + b$

$5/2 = (3/5)(-21/2) + b$

$5/2 = -63/10 + b$

$44/5 = b$

$y = 3/5x + 44/5$

• ⊥ of AC

midpoint $\left(\frac{-4-9}{2}, \frac{3+0}{2}\right)$

mdpt $\left(-13/2, 3/2\right)$

$m_{AC} = \frac{3-0}{-4-(-9)} = \frac{3}{5}$

$m_{\perp} = -5/3$

$y = mx + b$

$(3/2) = (-5/3)(-13/2) + b$

$3/2 = 65/6 + b$

$-28/3 = b$

$y = -5/3x - 28/3$

⊥AB $\begin{cases} y = 4x + 36 \\ y = 3/5x + 44/5 \end{cases}$ ⊥BC

$5(4x + 36 = 3/5x + 44/5)$

$20x + 180 = 3x + 44$

$17x = -136$

$x = -8$

$(-8, 4)$

⊥BC $\begin{cases} y = 3/5x + 44/5 \\ y = -5/3x - 28/3 \end{cases}$ ⊥AC

$15(3/5x + 44/5 = -5/3x - 28/3)$

$9x + 132 = -25x - 140$

$34x = -272$

$x = -8$

$(-8, 4)$

⊥AB $\begin{cases} y = 4x + 36 \\ y = -5/3x - 28/3 \end{cases}$ ⊥AC

$(4x + 36 = -5/3x - 28/3)3$

$12x + 108 = -5x - 28$

$17x = -136$

$x = -8$

$(-8, 4)$

now find r:

• using Point A: $r = \sqrt{(-8-(-4))^2 + (4-3)^2} = \sqrt{16+1} = \sqrt{17}$

• using Point B: $r = \sqrt{(-8-(-12))^2 + (4-5)^2} = \sqrt{16+1} = \sqrt{17}$

• using Point C: $r = \sqrt{(-8-(-9))^2 + (4-0)^2} = \sqrt{1+16} = \sqrt{17}$

$$(x+8)^2 + (y-4)^2 = 17$$

④ A(-16, -12) B(-16, 0) C(2, -18)

• ⊥ of AB

midpoint $\left(\frac{-16-16}{2}, \frac{-12+0}{2}\right)$

mdpt (-16, -6)

$m_{AB} = \frac{-12-0}{-16-(+16)} = \frac{-12}{0}$

$m_\perp = 0$

$y = mx + b$

$-6 = (0)(-16) + b$

$-6 = b$

$y = -6$

• ⊥ of BC

midpoint $\left(\frac{-16+2}{2}, \frac{0-18}{2}\right)$

mdpt (-7, -9)

$m_{BC} = \frac{0-(+18)}{-16-2} = \frac{18}{-18} = -1$

$m_\perp = 1$

$y = mx + b$

$-9 = (1)(-7) + b$

$-9 = -7 + b$

$-2 = b$

$y = x - 2$

• ⊥ of AC

midpoint $\left(\frac{-16+2}{2}, \frac{-12-18}{2}\right)$

mdpt (-7, -15)

$m_{AC} = \frac{-12-(+18)}{-16-2} = \frac{6}{-18} = -\frac{1}{3}$

$m_\perp = 3$

$y = mx + b$

$-15 = (3)(-7) + b$

$-15 = -21 + b$

$6 = b$

$y = 3x + 6$

⊥AB { $y = -6$
⊥BC { $y = x - 2$

$-6 = x - 2$

$-4 = x$

$(-4, -6)$

⊥BC { $y = x - 2$
⊥AC { $y = 3x + 6$

$x - 2 = 3x + 6$

$-8 = 2x$

$-4 = x$

$(-4, -6)$

⊥AB { $y = -6$
⊥AC { $y = 3x + 6$

$-6 = 3x + 6$

$-12 = 3x$

$-4 = x$

$(-4, -6)$

now find r:

• using Point A: $r = \sqrt{(-4+16)^2 + (-6+12)^2} = \sqrt{144+36} = \sqrt{180}$

• using Point B: $r = \sqrt{(-4+16)^2 + (-6-0)^2} = \sqrt{144+36} = \sqrt{180}$

• using Point C: $r = \sqrt{(-4-2)^2 + (-6+18)^2} = \sqrt{36+144} = \sqrt{180}$

$$(x+4)^2 + (y+6)^2 = 180$$

③ A(0, −8) B(1, −5) C(3, −9)

• ⊥ of AB

midpoint $\left(\frac{0+1}{2}, \frac{-8-5}{2}\right)$

mdpt $(1/2, -13/2)$

$m_{AB} = \frac{-8-(+5)}{0-1} = \frac{-3}{-1} = 3$

$m_{\perp} = -1/3$

$y = mx + b$

$-13/2 = (-1/3)(1/2) + b$

$-13/2 = -1/6 + b$

$-19/3 = b$

$y = -1/3\,x - 19/3$

• ⊥ of BC

midpoint $\left(\frac{1+3}{2}, \frac{-5-9}{2}\right)$

mdpt $(2, -7)$

$m_{BC} = \frac{-5-(+9)}{1-3} = \frac{4}{-2} = -2$

$m_{\perp} = 1/2$

$y = mx + b$

$-7 = (1/2)(2) + b$

$-7 = 1 + b$

$-8 = b$

$y = 1/2\,x - 8$

• ⊥ of AC

midpoint $\left(\frac{0+3}{2}, \frac{-8-9}{2}\right)$

mdpt $(3/2, -17/2)$

$m_{AC} = \frac{-8-(+9)}{0-3} = \frac{1}{-3}$

$m_{\perp} = 3$

$y = mx + b$

$-17/2 = (3)(3/2) + b$

$-17/2 = 9/2 + b$

$-13 = b$

$y = 3x - 13$

⊥AB $y = -1/3\,x - 19/3$

⊥BC $y = 1/2\,x - 8$

$(-1/3\,x - 19/3 = 1/2\,x - 8)\,6$

$-2x - 38 = 3x - 48$

$-5x = -10$

$x = 2$

$(2, -7)$

⊥BC $y = 1/2\,x - 8$

⊥AC $y = 3x - 13$

$(1/2\,x - 8 = 3x - 13)\,2$

$x - 16 = 6x - 26$

$10 = 5x$

$2 = x$

$(2, -7)$

⊥AB $y = -1/3\,x - 19/3$

⊥AC $y = 3x - 13$

$(-1/3\,x - 19/3 = 3x - 13)\,3$

$-x - 19 = 9x - 39$

$20 = 10x$

$2 = x$

$(2, -7)$

now find r:

• using Point A: $r = \sqrt{(2-0)^2 + (-7+8)^2} = \sqrt{4+1} = \sqrt{5}$

• using Point B: $r = \sqrt{(2-1)^2 + (-7+5)^2} = \sqrt{1+4} = \sqrt{5}$

• using Point C: $r = \sqrt{(2-3)^2 + (-7+9)^2} = \sqrt{1+4} = \sqrt{5}$

$$(x-2)^2 + (y+7)^2 = 5$$

⑥ A(12,4) B(2,−6) C(6,10)

• ⊥ of AB

midpoint $\left(\frac{12+2}{2}, \frac{4-6}{2}\right)$

mdpt (7, −1)

$m_{AB} = \frac{4-(-6)}{12-2} = \frac{10}{10} = 1$

$m_{\perp} = -1$

$y = mx + b$

$-1 = (-1)(7) + b$

$-1 = -7 + b$

$6 = b$

$y = -x + 6$

• ⊥ of BC

midpoint $\left(\frac{2+6}{2}, \frac{-6+10}{2}\right)$

mdpt (4, 2)

$m_{BC} = \frac{-6-10}{2-6} = \frac{-16}{-4} = 4$

$m_{\perp} = -1/4$

$y = mx + b$

$2 = (-1/4)(4) + b$

$2 = -1 + b$

$3 = b$

$y = -1/4x + 3$

• ⊥ of AC

midpoint $\left(\frac{12+6}{2}, \frac{4+10}{2}\right)$

mdpt (9, 7)

$m_{AC} = \frac{4-10}{12-6} = \frac{-6}{6} = -1$

$m_{\perp} = 1$

$y = mx + b$

$7 = (1)(9) + b$

$7 = 9 + b$

$-2 = b$

$y = x - 2$

$\perp AB \begin{cases} y = -x + 6 \\ \perp BC \end{cases} \begin{cases} y = -\frac{1}{4}x + 3 \end{cases}$

$(-x + 6 = -1/4x + 3)\,4$

$-4x + 24 = -x + 12$

$12 = 3x$

$4 = x$

$y = 2$

(4, 2)

$\perp BC \begin{cases} y = -1/4x + 3 \\ \perp AC \end{cases} \begin{cases} y = x - 2 \end{cases}$

$(-1/4x + 3 = x - 2)\,4$

$-x + 12 = 4x - 8$

$20 = 5x$

$4 = x$

$y = 2$

(4, 2)

$\perp AB \begin{cases} y = -x + 6 \\ + \perp AC \end{cases} \begin{cases} y = x - 2 \end{cases}$

$2y = 4$

$y = 2$

$x = 4$

(4, 2)

find r:

• using point A: $r = \sqrt{(4-12)^2 + (2-4)^2} = \sqrt{64 + 4} = \sqrt{68}$

• using point B: $r = \sqrt{(4-2)^2 + (2-(-6))^2} = \sqrt{4 + 64} = \sqrt{68}$

• using point C: $r = \sqrt{(4-6)^2 + (2-10)^2} = \sqrt{4 + 64} = \sqrt{68}$

$$(x-4)^2 + (y-2)^2 = 68$$