

## HM3 Midterm Review

①  $A = Pe^{rt}$   
 $A = 6746 e^{(.08)(19)}$   
 $\$30,844.23$

②  $A = P(1 + \frac{r}{n})^{nt}$   
 $A = 2953(1 + \frac{.06}{12})^{12 \cdot 13}$   
 $A = 2953(1.005)^{156}$   
 $\$6429.38$

③  $A = P(1 + \frac{r}{n})^{nt}$   
 $19976.27 = 4375(1 + \frac{.09}{2})^{2t}$   
 $4.566 = 1.045^{2t}$   
 $\log_{1.045} 4.566 = 2t$   
 $34.501 = 2t$   
 $t \approx 17.3 \text{ years}$

④  $A = Pe^{rt}$   
 $3380.08 = 1565 e^{.07t}$   
 $2.1598 = e^{.07t}$   
 $\ln 2.1598 = .07t$   
 $.777001 = .07t$   
 $t \approx 11 \text{ years}$

⑤  $A = Pe^{rt}$   
 $7779.35 = 5320 e^{r(19)}$   
 $1.46228 = e^{19r}$   
 $\ln 1.46228 = 19r$   
 $.379999 = 19r$   
 $.01999997 = r$   
 $r \approx 2\%$

⑥  $A = P(1 + \frac{r}{n})^{nt}$   
 $5142.19 = P(1 + \frac{.03}{4})^{4(6)}$   
 $5142.19 = P(1.0075)^{24}$   
 $5142.19 = P(1.19641)$   
 $P = \$4298$

⑦ 
$$\begin{array}{r|rrrr} 6 & 5 & -30 & -2 & 5 \\ + \downarrow & & 30 & 0 & -12 \\ \hline & 5 & 0 & -2 & -7 \end{array}$$
  
 $5x^2 - 2 + x - 6$

⑧ 
$$\begin{array}{r|rrrr} -5 & 6 & 28 & -11 & 0 \\ + \downarrow & & -30 & 10 & 5 \\ \hline & 6 & -2 & -1 & 5 \end{array}$$
  
 $6x^2 - 2x - 1 + x - 5$

$$\begin{array}{r}
 3x^2 + 5x + 4 \\
 \textcircled{9} \quad 5x - 6 \overline{) 15x^3 + 7x^2 - 10x - 19} \\
 \underline{-15x^3 + 18x^2} \phantom{-19} \\
 25x^2 - 10x \phantom{-19} \\
 \underline{-25x^2 + 30x} \phantom{-19} \\
 20x - 19 \\
 \underline{-20x + 24} \\
 5
 \end{array}$$

$$3x^2 + 5x + 4 + \frac{5}{5x-6}$$

$$\begin{array}{r}
 \textcircled{10} \quad 1 \overline{) 3 \quad 8 \quad -3 \quad -6 \quad 9} \\
 + \downarrow \quad 3 \quad 11 \quad 8 \quad 2 \\
 \hline
 3 \quad 11 \quad 8 \quad 2 \quad 11
 \end{array}$$

$$3x^3 + 11x^2 + 8x + 2 + \frac{11}{x-1}$$

$$\begin{aligned}
 \textcircled{11} \quad 2^{3-3n} &= 64 \\
 \log_2 64 &= 3-3n \\
 6 &= 3-3n \\
 3 &= -3n
 \end{aligned}$$

$$-1 = n$$

$$\begin{aligned}
 \textcircled{12} \quad 5^{-n+3} &= 125 \\
 \log_5 125 &= -n+3 \\
 3 &= -n+3 \\
 0 &= -n
 \end{aligned}$$

$$0 = n$$

$$\begin{aligned}
 \textcircled{13} \quad 625^x &= 125^{-x-2} \\
 (5^4)^x &= (5^3)^{-x-2} \\
 4x &= -3x-6
 \end{aligned}$$

$$7x = -6$$

$$x = -6/7$$

$$\begin{aligned}
 \textcircled{14} \quad 4^{-2n+1} \cdot 2^2 &= \left(\frac{1}{64}\right)^{2n} \\
 (2^2)^{-2n+1} \cdot 2^2 &= (2^{-6})^{2n} \\
 2^{-4n+2} \cdot 2^2 &= 2^{-12n} \\
 2^{-4n+4} &= 2^{-12n}
 \end{aligned}$$

$$-4n+4 = -12n$$

$$4 = -8n$$

$$-1/2 = n$$

$$\textcircled{15} \quad \frac{8^{2x}}{16^{-x}} = 2^3$$

$$\frac{(2^3)^{2x}}{(2^4)^{-x}} = 2^3$$

$$\frac{2^{6x}}{2^{-4x}} = 2^3$$

$$2^{10x} = 2^3$$

$$10x = 3$$

$$x = 3/10$$

$$\begin{aligned} (16) \quad 16 \cdot 64^{-a} &= 16^{-a} \\ 4^2 \cdot (4^3)^{-a} &= (4^2)^{-a} \\ 4^2 \cdot 4^{-3a} &= 4^{-2a} \\ 4^{-3a+2} &= 4^{-2a} \\ -3a+2 &= -2a \end{aligned}$$

$$2 = a$$

$$\begin{aligned} (17) \quad -3 \cdot 10^{-6v} &= -32 \\ 10^{-6v} &= 32/3 \\ \log_{10} 32/3 &= -6v \\ 1.0280 &= -6v \end{aligned}$$

$$-.1713 = v$$

$$\begin{aligned} (18) \quad 4 \cdot 8^{x+4} &= 18 \\ 8^{x+4} &= 9/2 \\ \log_8 9/2 &= x+4 \\ .7233 &= x+4 \end{aligned}$$

$$-3.2767 = x$$

$$\begin{aligned} (19) \quad 9e^m - 5 &= 67 \\ 9e^m &= 72 \\ e^m &= 8 \\ \ln 8 &= m \end{aligned}$$

$$2.0794 = m$$

$$\begin{aligned} (20) \quad -e^n - 5.6 &= -59 \\ -e^n &= -53.4 \\ e^n &= 53.4 \\ \ln 53.4 &= n \end{aligned}$$

$$3.9778 = n$$

$$\begin{aligned} (21) \quad \ln(4b+3) &= \ln(5b+2) \\ 4b+3 &= 5b+2 \end{aligned}$$

$$1 = b$$

$$\begin{aligned} (23) \quad \log_{18}(x^2-46) &= \log_{18}(2x+2) \\ x^2-46 &= 2x+2 \\ x^2-2x-48 &= 0 \\ (x-8)(x+6) &= 0 \end{aligned}$$

$$x = 8$$

$$x = -6$$

$$\begin{aligned} (22) \quad \ln(5n-4) &= \ln 16 \\ 5n-4 &= 16 \\ 5n &= 20 \end{aligned}$$

$$n = 4$$

$$\begin{aligned} (24) \quad \log_{19}(v^2-7v) &= \log_{19}(16-v) \\ v^2-7v &= 16-v \\ v^2-6v-16 &= 0 \\ (v-8)(v+2) &= 0 \end{aligned}$$

$$v = 8 \quad v = -2$$

$$\begin{aligned} (25) \quad \log_3(x+2) - 9 &= -10 \\ \log_3(x+2) &= -1 \\ 3^{-1} &= x+2 \\ 1/3 &= x+2 \end{aligned}$$

$$-5/3 = x$$

$$\textcircled{26} \log_2(v+5) - 2 = -4$$

$$\log_2(v+5) = -2$$

$$2^{-2} = v+5$$

$$1/4 = v+5$$

$$\boxed{-19/4 = v}$$

$$\textcircled{28} \ln 3 - \ln(x+8) = 5$$

$$\ln \frac{3}{x+8} = 5$$

$$e^5 = \frac{3}{x+8}$$

$$(x+8)e^5 = 3$$

$$x+8 = \frac{3}{e^5}$$

$$x = \frac{3}{e^5} - 8$$

$$\boxed{x = -7.9798}$$

$$\textcircled{30} \ln 2x^2 - \ln 6 = \ln 27$$

$$\ln \frac{2x^2}{6} = \ln 27$$

$$\frac{x^2}{3} = 27$$

$$x^2 = 81$$

$$\boxed{x = \pm 9}$$

$$\textcircled{32} \log_8(x^2+2) - \log_8 3 = \log_8 34$$

$$\log_8 \frac{x^2+2}{3} = \log_8 34$$

$$\frac{x^2+2}{3} = 34$$

$$x^2+2 = 102$$

$$x^2 = 100$$

$$\boxed{x = \pm 10}$$

$$\textcircled{27} \ln 2x^2 - \ln 2 = 4$$

$$\ln \frac{2x^2}{2} = 4$$

$$e^4 = \frac{2x^2}{2}$$

$$e^4 = x^2$$

$$\pm \sqrt{e^4} = x$$

$$\boxed{\pm 7.389 = x}$$

$$\textcircled{29} \ln(x+16) + \ln x = \ln 80$$

$$\ln(x^2+16x) = \ln 80$$

$$x^2+16x = 80$$

$$x^2+16x-80 = 0$$

$$(x+20)(x-4) = 0$$

$$\cancel{x = -20} \quad \boxed{x = 4}$$

$$\textcircled{31} \ln 2x^2 - \ln 8 = 4$$

$$\ln \frac{2x^2}{8} = 4$$

$$e^4 = \frac{x^2}{4}$$

$$4e^4 = x^2$$

$$\pm \sqrt{4e^4} = x$$

$$\boxed{\pm 14.778 = x}$$

$$\textcircled{33} \log_5 8 - \log_5(x-9) = 1$$

$$\log_5 \frac{8}{x-9} = 1$$

$$5^1 = \frac{8}{x-9}$$

$$5(x-9) = 8$$

$$x-9 = 8/5$$

$$x = 8/5 + 9$$

$$\boxed{x = 53/5}$$

$$\textcircled{34} \log_5(x+5) - \log_5 x = \log_5 61$$

$$\log_5 \frac{x+5}{x} = \log_5 61$$

$$\frac{x+5}{x} = 61$$

$$x+5 = 61x$$

$$5 = 60x$$

$$\frac{5}{60} = x$$

$$\frac{1}{12} = x$$

$$\textcircled{35} f(x) = x^4 - 3x^2 + 4$$

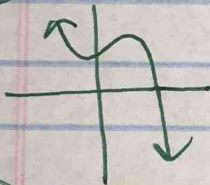


absolute min at  $x = -1.22$ ,

$x = 1.22$

relative max at  $x = 0$

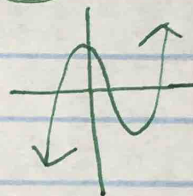
$$\textcircled{36} f(x) = -x^3 + 3x^2 + 3$$



rel. min at  $x = 0$

rel. max at  $x = 2$

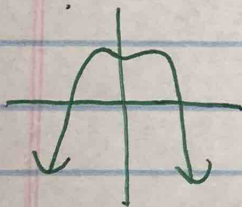
$$\textcircled{37} f(x) = x^3 - 4x^2 + 6$$



Inc:  $(-\infty, 0)$   $(2.67, \infty)$

Dec:  $(0, 2.67)$

$$\textcircled{38} f(x) = -x^4 + 2x^2 + 3$$



absolute max

at  $x = -1, x = 1$

rel. min at  $x = 0$

$$(39) f(x) = \frac{5}{8}x - \frac{25}{8}$$

$$y = \frac{5}{8}x - \frac{25}{8}$$

$$x = \frac{5}{8}y - \frac{25}{8}$$

$$x + \frac{25}{8} = \frac{5}{8}y$$

$$8x + 25 = 5y$$

$$\frac{8x + 25}{5} = f'(x)$$

$$\text{or } \frac{8}{5}x + 5 = f'(x)$$

$$(41) f(x) = \frac{3}{x+2}$$

$$y = \frac{3}{x+2}$$

$$x = \frac{3}{y+2}$$

$$(y+2)(x) = 3$$

$$y+2 = \frac{3}{x}$$

$$y = \frac{3}{x} - 2$$

$$f'(x) = \frac{3}{x} - 2$$

$$(40) f(x) = \frac{-4+x}{4}$$

$$y = \frac{-4+x}{4}$$

$$x = \frac{-4+y}{4}$$

$$4x = -4 + y$$

$$4x + 4 = f'(x)$$

$$(42) g(x) = \sqrt[3]{x+1} - 1$$

$$y = \sqrt[3]{x+1} - 1$$

$$x = \sqrt[3]{y+1} - 1$$

$$x+1 = \sqrt[3]{y+1}$$

$$(x+1)^3 = y+1$$

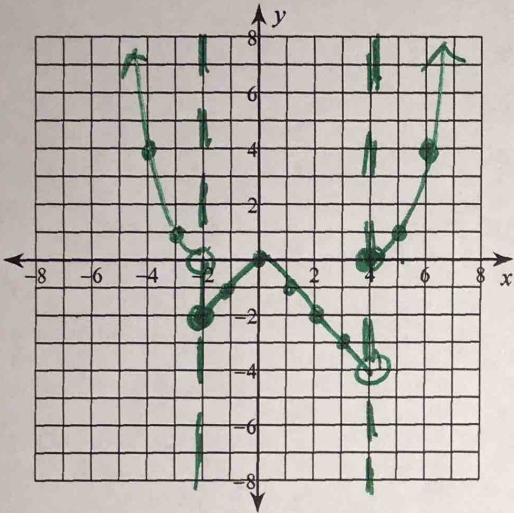
$$(x+1)^3 - 1 = y$$

$$(x+1)^3 - 1 = f'(x)$$

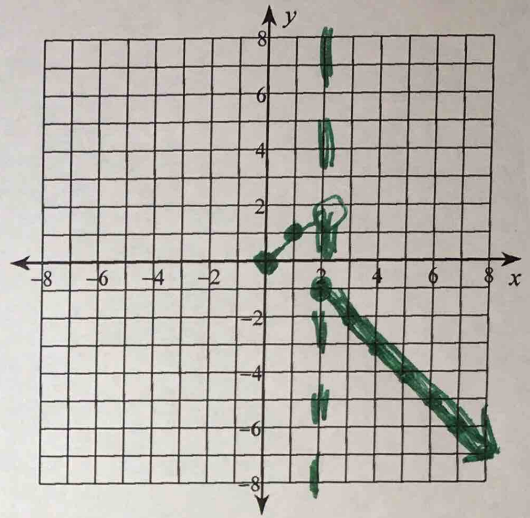
\* 43-46 on graph paper \*

Sketch the graph of each function.

$$43) f(x) = \begin{cases} (x+2)^2, & x < -2 \\ -|x|, & -2 \leq x < 4 \\ (x-4)^2, & x \geq 4 \end{cases}$$

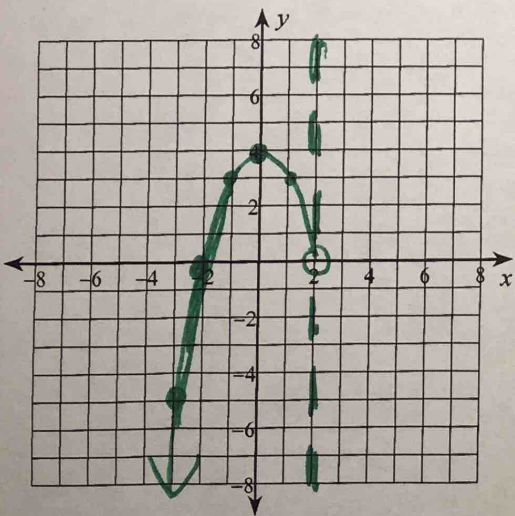


$$44) h(x) = \begin{cases} \sqrt{x}, & x < 2 \\ -x + 1, & x \geq 2 \end{cases}$$

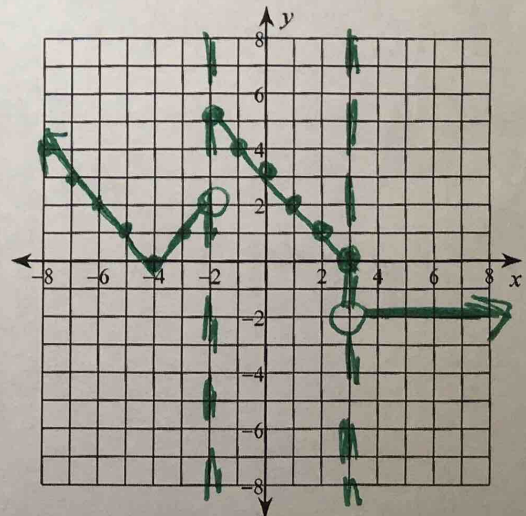


$$45) g(x) = \begin{cases} 4 - x^2, & x < 2 \\ -x - 2, & x > 2 \end{cases}$$

*Handwritten note:  $-x^2 + 4$  with an arrow pointing to the first part of the function.*



$$46) f(x) = \begin{cases} |x+4|, & x < -2 \\ |x-3|, & -2 \leq x \leq 3 \\ -2, & x > 3 \end{cases}$$



47)  $f(x) = x^3 + 4x^2 + 4x + 16$

3 roots

$p=16$        $q=1$   
 $1, 2, 4, 8, 16$        $1$

$f(x) = x^3 + 4x^2 + 4x + 16$

P      0 changes

$f(-x) = (-x)^3 + 4(-x)^2 + 4(-x) + 16$

$f(-x) = -x^3 + 4x^2 - 4x + 16$

$N \rightarrow P \rightarrow N \rightarrow P$   
 3 changes

possible rational roots:  
 $\pm 1, 2, 4, 8, 16$

<del>+R</del>	0	0
<del>-R</del>	3	1
imag/irr	0	2

\*to find zeros just go straight to your calculator!

calc table:  $(-4, 0)$      $-4 \mid 1 \quad 4 \quad 4 \quad 16$   
 $\quad \quad \quad + \downarrow -4 \quad 0 \quad -16$   


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 $\quad \quad \quad 1 \quad 0 \quad 4 \quad 0$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

Roots:

$x = -4, \pm 2i$

48)  $f(x) = x^3 + 4x^2 + 2x + 8$

3 roots

$p=8$        $q=1$   
 $1, 2, 4, 8$        $1$

$f(x) = x^3 + 4x^2 + 2x + 8$

P      0 changes

$f(-x) = (-x)^3 + 4(-x)^2 + 2(-x) + 8$

$f(-x) = -x^3 + 4x^2 - 2x + 8$

$N \rightarrow P \rightarrow N \rightarrow P$

3 changes

possible rational roots:  
 $\pm 1, 2, 4, 8$

<del>+R</del>	0	0
<del>-R</del>	3	1
imag/irr	0	2

calc table:  $(-4, 0)$      $-4 \mid 1 \quad 4 \quad 2 \quad 8$   
 $\quad \quad \quad + \downarrow -4 \quad 0 \quad -8$   


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 $\quad \quad \quad 1 \quad 0 \quad 2 \quad 0$

$x^2 + 2 = 0$

$x^2 = -2$

$x = \pm i\sqrt{2}$

Roots:

$x = -4, \pm i\sqrt{2}$



49)  $f(x) = x^3 + 5x^2 + 3x + 15$

3 roots

$p = 15$   $q = 1$   
 $\pm 1, 3, 5, 15$   $\pm 1$

$f(x) = x^3 + 5x^2 + 3x + 15$

0 changes

$f(-x) = (-x)^3 + 5(-x)^2 + 3(-x) + 15$

$f(-x) = -x^3 + 5x^2 - 3x + 15$

$N \rightarrow P \rightarrow N \rightarrow P$  3 changes

calc table:  $(-5, 0)$

$$\begin{array}{r|rrrr} -5 & 1 & 5 & 3 & 15 \\ + \downarrow & & -5 & 0 & -15 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

$x^2 + 3 = 0$

$x^2 = -3$

$x = \pm i\sqrt{3}$

Roots:  $x = -5, \pm i\sqrt{3}$

possible roots:  $\pm 1, 3, 5, 15$

+R	0	0
-R	3	1
imag/irr	0	2

50)  $f(x) = x^4 - 7x^2 - 18$

4 roots

$p = 18$   $q = 1$   
 $\pm 1, 2, 3, 6, 9, 18$   $\pm 1$

$f(x) = x^4 - 7x^2 - 18$

$P \rightarrow N$  1 change

$f(-x) = (-x)^4 - 7(-x)^2 - 18$

$f(-x) = x^4 - 7x^2 - 18$

$P \rightarrow N$  1 change

$3 \mid 1 \ 0 \ -7 \ 0 \ -18$

$+ \downarrow 3 \ 9 \ 6 \ 18$

$1 \ 3 \ 2 \ 6 \ 0$

$-3 \mid 1 \ 3 \ 2 \ 6$

$+ \downarrow -3 \ 0 \ -6$

$1 \ 0 \ 2 \ 0$

possible roots:  $\pm 1, 2, 3, 6, 9, 18$

+R	1
-R	1
imag/irr	2

calc table:  $(-3, 0)$   $(3, 0)$

$x^2 + 2 = 0$

$x^2 = -2$

$x = \pm i\sqrt{2}$

Roots:  $x = -3, 3, \pm i\sqrt{2}$