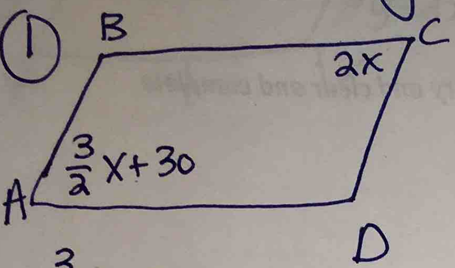
HM3 Geometry Review

① 

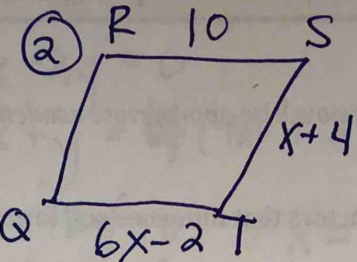
$\frac{3}{2}x+30=2x$

$30=\frac{1}{2}x$

$60=x$

$m\angle A=\frac{3}{2}(60)+30$

$\boxed{m\angle A=120^\circ}$

② 

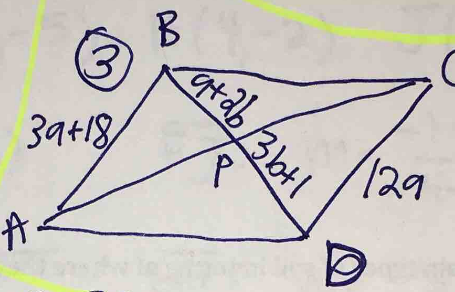
$6x-2=10$

$6x=12$

$x=2$

$ST=2+4$

$\boxed{\overline{QR}=6}$

③ 

$3a+18=12a$

$18=9a$

$\boxed{a=2}$

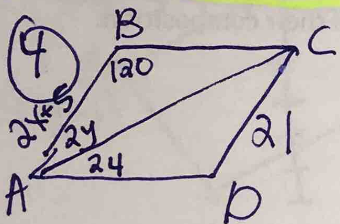
$a+2b=3b+1$

$2+2b=3b+1$

$\boxed{1=b}$

$\overline{DB}=2+2(1)+3(1)+1$

$\boxed{\overline{DB}=8}$

④ 

$2x+5=21$

$2x=16$

$\boxed{x=8}$

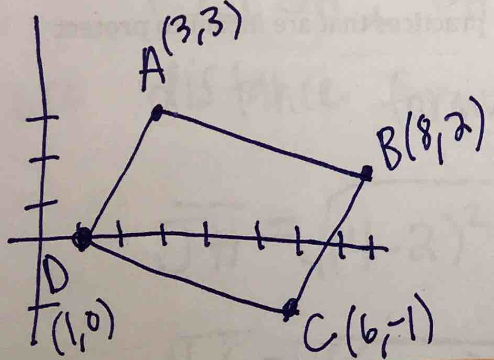
$2y+24+120=180$

$2y+144=180$

$2y=36$

$\boxed{y=18}$

⑤ A(3,3) B(8,2) C(6,−1) D(1,0)



$\overline{AB}: m=\frac{3-2}{3-8}=\frac{1}{-5}$

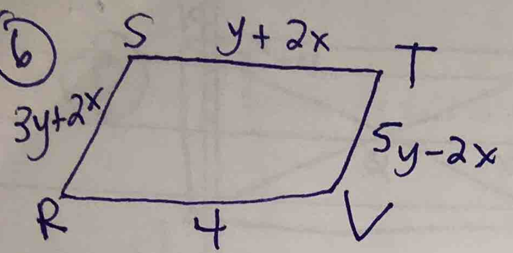
$\overline{DC}: m=\frac{0--1}{1-6}=\frac{1}{-5}$ } parallel

$\overline{AD}: m=\frac{3-0}{3-1}=\frac{3}{2}$

$\overline{BC}: m=\frac{2--1}{8-6}=\frac{3}{2}$ } parallel

Both pairs of opposite sides are parallel, so this <u>is</u> a parallelogram

⑥



$4 = y + 2x$ ☆

$\cancel{3y+2x = 5y-2x}$

$3y + 2x = 5y - 2x$

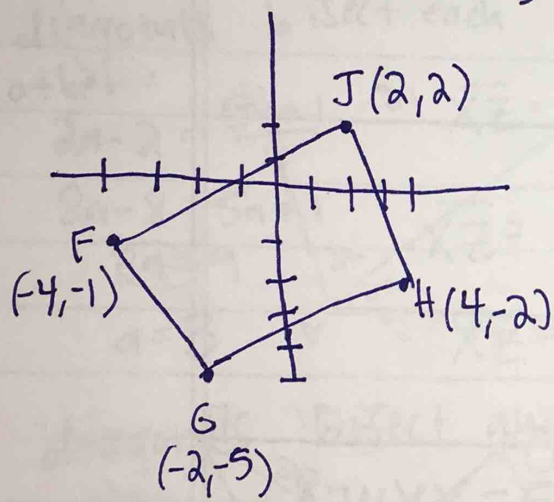
$4x - 2y = 0$ ☆

$$\begin{cases} 4x - 2y = 0 \\ 2x + y = 4 \text{ (mult. 2)} \end{cases}$$

$$\begin{array}{r} 4x - 2y = 0 \\ + \;\; 4x + 2y = 8 \\ \hline 8x \;\;\;\;\; = 8 \end{array}$$

$\boxed{x = 1}$ $\boxed{y = 2}$

⑦ $F(-4,-1)$ $G(-2,-5)$ $H(4,-2)$ $J(2,2)$



$\overline{FJ}$: $m = \dfrac{-1-2}{-4-2} = \dfrac{-3}{-6} = \dfrac{1}{2}$

$\overline{GH}$: $m = \dfrac{-5--2}{-2-4} = \dfrac{-3}{-6} = \dfrac{1}{2}$

parallel

$\overline{JH}$: $m = \dfrac{2--2}{2-4} = \dfrac{4}{-2} = -2$

$\overline{FG}$: $m = \dfrac{-1--5}{-4--2} = \dfrac{4}{-2} = -2$

parallel

☆ the slopes are OPPOSITE RECIPROCALS ☆ → so the lines are PERPENDICULAR!

$\overline{FJ} \perp \overline{JH}$, $\overline{GH} \perp \overline{GF}$ ⇒ it is either a rectangle or square

use distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\overline{JH} = \sqrt{(4-2)^2 + (-2-2)^2} = \sqrt{4+16} = \sqrt{20}$

$\overline{HG} = \sqrt{(4--2)^2 + (-2--5)^2} = \sqrt{36+9} = \sqrt{45}$

all sides are not congruent!

RECTANGLE!

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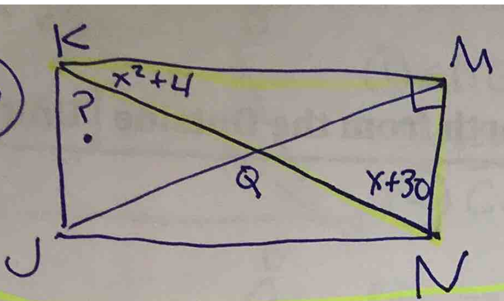

we know rectangles have right angles, so in this $\triangle$:

$$x^2+4+x+30+90=180$$
$$x^2+x+124=180$$
$$x^2+x-56=0$$
$$(x+8)(x-7)=0$$
$$\boxed{x=-8,\ x=7}$$

so $\angle NKM = 68°$ or $53°$

$\boxed{\text{so } \angle JKN = 22° \text{ or } 37°}$

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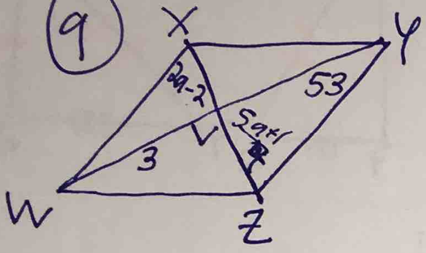


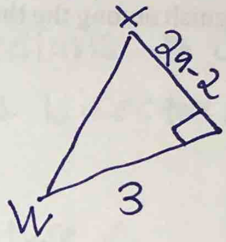
$\boxed{\begin{aligned} m\angle XYW &= 53° \\ \overline{XZ} &= 8 \\ XW &= 5 \end{aligned}}$

diagonals bisect each other:

$$2a-2 = \frac{5a+1}{4}$$
$$8a-8 = 5a+1$$
$$3a = 9$$
$$a = 3$$

$$\overline{XZ} = 2(3)-2+\frac{5(3)+1}{4}$$
$$\overline{XZ} = 6-2+4$$
$$\boxed{\overline{XZ} = 8}$$

we know the diagonals create right angles:



$\leftarrow 2(3)-2 = 4$

$$3^2+4^2=(XW)^2$$
$$9+16=(XW)^2$$
$$25=(XW)^2$$
$$\boxed{5=XW}$$

diagonals bisect angles, so if $m\angle WYZ = 53°$, then $\boxed{m\angle WYX = 53°}$

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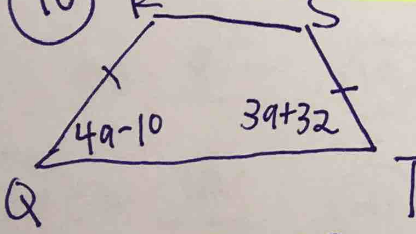

base angles are $\cong$

$$4a-10 = 3a+32$$
$$a = 42$$
$$\angle Q = 4(42)-10$$
$$\angle Q = 158$$

$\angle Q$ & $\angle R$ are supp.

$$\angle R = 180-158$$
$$\boxed{\angle R = 22°}$$

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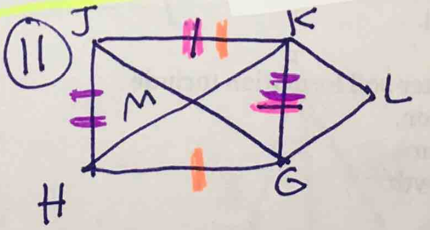
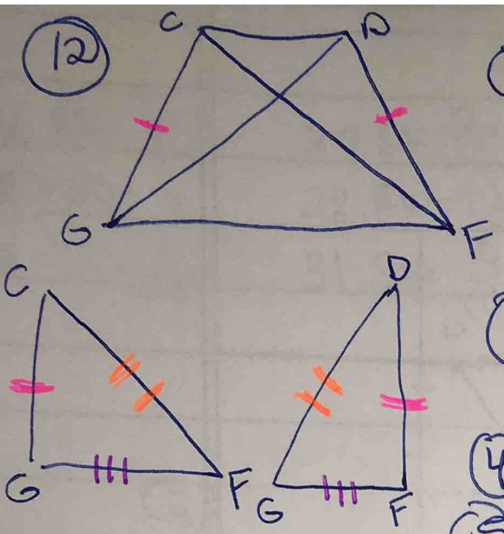


1. $\triangle LGK \cong \triangle MJK$ — 1. given
2. $\overline{GK} \cong \overline{JK}$ — 2. CPCTC [1]
3. GHJK is a parallelogram — 3. given
4. $\overline{JK} \cong \overline{HG}$, $\overline{KG} \cong \overline{JH}$ — 4. in a parallelogram, opp. sides are $\cong$
5. $\overline{JK} \cong \overline{GK} \cong \overline{JH} = \overline{HG}$ — 5. substitution [2, 4]
6. GHJK is a rhombus — 6. def. rhombus [3, 5]

(12)



Statements	Reasons
① CDGF is an isosceles trapezoid	① Given
② $\overline{CG} \cong \overline{DF}$	② Def. isosceles trapezoid (legs are ≅) [1]
③ $\overline{DG} \cong \overline{CF}$	③ Def. isosceles trapezoid (diagonals are ≅) [1]
④ $\overline{GF} \cong \overline{GF}$	④ Reflexive property
⑤ $\triangle DGF \cong \triangle CFG$	⑤ SSS [2, 3, 4]
⑥ $\angle DGF \cong \angle CFG$	⑥ CPCTC [5]

(13) angle bisectors → INCENTER (14) medians → CENTROID

(15) altitudes → ORTHOCENTER (16) ⊥ bisectors → CIRCUMCENTER

(17)

	Acute △	Obtuse △	Right △
circumcenter	inside	outside	on
incenter	inside	inside	inside
centroid	inside	inside	inside
orthocenter	inside	outside	on

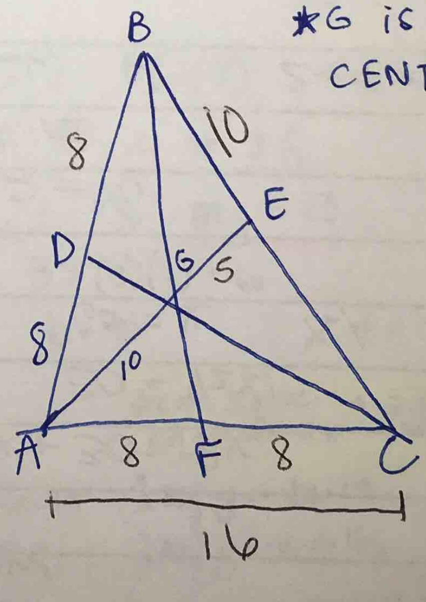
(18.) DB = 8 (19.) EA = 15

(20.) CG = 12 (21.) BA = 16

(22). GE = 5 (23). GD = 6

(24) BC = 20 (25). AF = 8

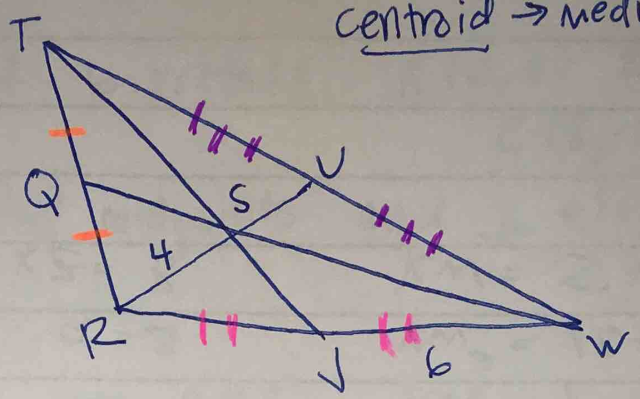
★G is the CENTROID ★medians!

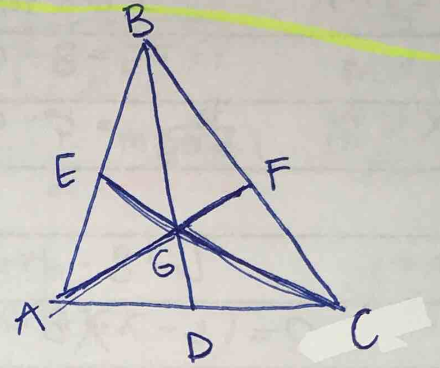


26. RV = $\underline{6}$
28. RU = $\underline{6}$
30. TS = $\underline{6}$

27. SU = $\underline{2}$
28. RW = $\underline{12}$
31. SV = $\underline{3}$

centroid → medians





(32) $FG = x+8$ $\quad GA = 6x-4$

2 to 1 ratio

$2(x+8) = 6x-4$

$x+8 = 3x-2$

$10 = 2x$

$\boxed{5 = x}$

(33) $CG = 3y+7$ $\quad CE = 6y$

the "2 to 1 ratio" means that CG is 2/3 of CE

$3y+7 = 2/3(6y)$

$3y+7 = 4y$

$\boxed{7 = y}$

(34) ORTHOCENTER → altitudes $\quad A(3,7) \quad B(1,3) \quad C(9,5)$

• altitude from A ⊥ to BC

$(3,7)$ $\quad m_{BC} = 2/8 = 1/4$

$m_\perp = -4$

$y = mx+b$ $\quad 7 = (-4)(3)+b$

$19 = b$

$y = -4x+19$

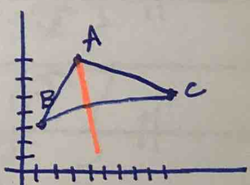
• altitude from B ⊥ to AC

$(1,3)$ $\quad m_{AC} = -1/3$ $\quad m_\perp = 3$

$y = mx+b$ $\quad 3 = (3)(1)+b$

$b = 0$

$y = 3x$



altitude from C ⊥ to AB

$(9,5)$ $\quad m_{AB} = 2$ $\quad m_\perp = -1/2$

$y = mx+b$ $\quad 5 = (-1/2)(9)+b$

$5 = -9/2 + b$

$19/2 = b$

$y = -1/2x + 19/2$

★can solve a system using any 2 equations★

$\perp BC \begin{cases} y = -4x+19 \end{cases}$
$\perp AC \begin{cases} y = 3x \end{cases}$

$-4x+19 = 3x$

$19 = 7x$

$x = 19/7$

$\boxed{\left(\frac{19}{7}, \frac{57}{7}\right)}$

$\perp BC \begin{cases} y = -4x+19 \end{cases}$
$\perp AB \begin{cases} y = -1/2x + 19/2 \end{cases}$

$-4x+19 = -1/2x + 19/2$

$-8x+38 = -x+19$

$19 = 7x$

$x = 19/7$

$\boxed{(19/7, \frac{57}{7})}$

$\perp AC \begin{cases} y = 3x \end{cases}$
$\perp AB \begin{cases} y = -1/2x + 19/2 \end{cases}$

$3x = -1/2x + 19/2$

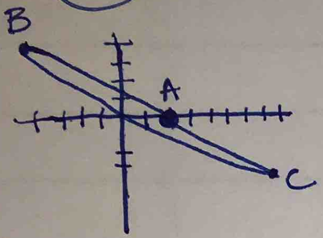
$6x = -x + 19$

$7x = 19$

$x = 19/7$

$\boxed{(19/7, 57/7)}$

35 **CIRCUMCENTER** $\perp$ bisectors $A(2,0)$ $B(-4,4)$ $C(8,-2)$



★need to find midpoints and $\perp$ slopes★

• $\perp$ bisector of AB

midpoint $(-1, 2)$

$m_{AB} = \frac{4}{-6} = \frac{-2}{3}$ $\quad m_{\perp} = \frac{3}{2}$

$y = mx + b$

$2 = (3/2)(-1) + b$

$2 = -3/2 + b$

$7/2 = b$

$y = 3/2\,x + 7/2$

• $\perp$ bisector of BC

midpoint $(2, 1)$

$m_{BC} = \frac{-6}{12} = -\frac{1}{2}$ $\quad m_{\perp} = 2$

$y = mx + b$

$1 = (2)(2) + b$

$1 = 4 + b$

$-3 = b$

$y = 2x - 3$

• $\perp$ bisector of AC

midpoint $(5, -1)$

$m_{AC} = \frac{-2}{6} = \frac{-1}{3}$ $\quad m_{\perp} = 3$

$y = mx + b$

$-1 = (3)(5) + b$

$-1 = 15 + b$

$-16 = b$

$y = 3x - 16$

★now use any 2 of the above 3 equations to solve★

$\perp AB$ $y = 3/2\,x + 7/2$
$\perp BC$ $y = 2x - 3$

$3/2\,x + 7/2 = 2x - 3$

$3x + 7 = 4x - 6$

$13 = x$

$(13, 23)$

$\perp BC$ $y = 2x - 3$
$\perp AC$ $y = 3x - 16$

$2x - 3 = 3x - 16$

$13 = x$

$(13, 23)$

$\perp AB$ $y = 3/2\,x + 7/2$
$\perp AC$ $y = 3x - 16$

$3/2\,x + 7/2 = 3x - 16$

$3x + 7 = 6x - 32$

$39 = 3x$

$13 = x$

$(13, 23)$

(36) centroid → medians $A(3,6)$ $B(4,7)$ $C(-5,-2)$

$$\left(\frac{3+4+-5}{3} , \frac{6+7+-2}{3} \right)$$

$$\boxed{\left(\frac{2}{3} , \frac{11}{3} \right)}$$