# Finding Roots by Hand Day 2 : 21, 23, 28, 30, 38, 44

(21) $x^3-4x^2-3x+18=0$ should have 3 roots

$p=18$ $q=1$

1, 2, 3, 6, 9, 18 1

possible rational roots: $\pm$ 1, 2, 3, 6, 9, 18

$f(x)=x^3-4x^2-3x+18$

P → N → P

2 changes

2 or 0 $+\mathbb{R}$

$f(-x)=(-x)^3-4(-x)^2-3(-x)+18$

$f(-x)=-x^3-4x^2+3x+18$

N → P

1 change

1 $-\mathbb{R}$

| | | |
|---|---|---|
| $+\mathbb{R}$ | 2 | 0 |
| $-\mathbb{R}$ | 1 | 1 |
| imag/irr | 0 | 2 |

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -3 & 18 \\ + & \downarrow & -2 & 12 & -18 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$x^2-6x+9=0$

$(x-3)(x-3)=0$

DOUBLE ROOT $x=3$

$$\boxed{\text{ROOTS: } x=-2,\ 3 \text{ (multiplicity 2)}}$$

23 $x^3-5x^2+4x-20$ 3 roots

$p=20$ $\quad$ $q=1$

1, 2, 4, 5, 10, 20 $\quad$ 1

possible rational roots: $\pm 1, 2, 4, 5, 10, 20$

$f(x)=x^3-5x^2+4x-20$

P → N → P → N

3 changes

3 or 1 $+\mathbb{R}$

$f(-x)=(-x)^3-5(-x)^2+4(-x)-20$

$f(-x)=-x^3-5x^2-4x-20$

N

0 changes

0 $-\mathbb{R}$

| | | |
|---|---|---|
| $+\mathbb{R}$ | 3 | 1 |
| $-\mathbb{R}$ | 0 | 0 |
| imag/irr | 0 | 2 |

$$\begin{array}{r|rrrr} 5 & 1 & -5 & 4 & -20 \\ + & \downarrow & 5 & 0 & 0 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2+4=0$

$x^2=-4$

$x=\pm\sqrt{-4}$

$x=\pm 2i$

ROOTS:

$\boxed{x=5, \pm 2i}$

28 $f(x) = x^3 - x^2 - 2x + 8$ 3 roots

$p = 8$ $q = 1$ possible rational roots: $\pm 1, 2, 4, 8$

1, 2, 4, 8 1

$f(x) = x^3 - x^2 - 2x + 8$

P → N → P

2 changes

2 or 0 +ℝ

$f(-x) = (-x)^3 - (-x)^2 - 2(-x) + 8$

$f(-x) = -x^3 - x^2 + 2x + 8$

N → P

1 change

1 −ℝ

| | | |
|---|---|---|
| +ℝ | 2 | ~~0~~ |
| −ℝ | 1 | 1 |
| imag/irr | 0 | 2 |

$$\begin{array}{r|rrrr} -2 & 1 & -1 & -2 & 8 \\ + & \downarrow & -2 & 6 & -8 \\ \hline & 1 & -3 & 4 & 0 \end{array}$$

$x^2 - 3x + 4 = 0$

$a = 1$
$b = -3$
$c = 4$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{-7}}{2}$$

$$x = \frac{3 \pm i\sqrt{7}}{2}$$

ROOTS:

$$\boxed{x = -2, \frac{3 \pm i\sqrt{7}}{2}}$$

(30) $2x^3+5x^2+6x+2=0$ should have 3 roots

$p=2$   $q=2$

$1, 2$   $1, 2$

possible rational roots: $\pm 1, \frac{1}{2}, 2$

$f(x)=2x^3+5x^2+6x+2=0$

P

0 changes

0 +ℝ

$f(-x)=2(-x)^3+5(-x)^2+6(-x)+2$

$f(-x)=-2x^3+5x^2-6x+2$

$N \to P \to N \to P$

3 changes

3 or 1 −ℝ

| | | |
|---|---|---|
| +ℝ | 0 | 0 |
| −ℝ | 3 | 1 |
| imag/irr | 0 | 2 |

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 5 & 6 & 2 \\ + & \downarrow & -1 & -2 & -2 \\ \hline & 2 & 4 & 4 & 0 \end{array}$$

$2x^2+4x+4=0$

★can ÷ by 2 to make solving easier★

$x^2+2x+2=0$

$x^2+2x+\underline{1}=-2+\underline{1}$

$(x+1)^2=-1$

$x+1=\pm\sqrt{-1}$

$x+1=\pm i$

$x=-1\pm i$

$$\boxed{\text{ROOTS: } x=-\frac{1}{2}, -1\pm i}$$

(38) $x^3 - 8x + 32 = 0$ 3 roots

$p = 32$ $q = 1$

$1, 2, 4, 8, 16, 32$ / $1$

possible rational roots: $\pm 1, 2, 4, 8, 16, 32$

$f(x) = x^3 - 8x + 32$

P → N → P

2 changes

2 or 0 +IR

$f(-x) = (-x)^3 - 8(-x) + 32$

$f(-x) = -x^3 + 8x + 32$

N → P

1 change

1 -IR

| | | |
|---|---|---|
| +IR | 2 | 0 |
| -IR | 1 | 1 |
| imag/irr | 0 | 2 |

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -8 & 32 \\ + & \downarrow & -4 & 16 & -32 \\ \hline & 1 & -4 & 8 & 0 \end{array}$$

$x^2 - 4x + 8 = 0$

$x^2 - 4x + \underline{4} = -8 + \underline{4}$

$(x-2)^2 = -4$

$x - 2 = \pm\sqrt{-4}$

$x - 2 = \pm 2i$

$x = 2 \pm 2i$

ROOTS:

$\boxed{x = -4,\ 2 \pm 2i}$

44) $x^4+x^2-20=0$ 4 roots

$p=20$ $q=1$

$1,2,4,5,10,20$ $1$

possible rational roots: $\pm 1, 2, 4, 5, 10, 20$

$f(x)=x^4+x^2-20$

$P \longrightarrow N$

1 change

1 +R

$f(-x)=(-x)^4+(-x)^2-20$

$f(-x)=x^4+x^2-20$

$P \longrightarrow N$

1 change

1 -R

| | |
|---|---|
| +R | 1 |
| -R | 1 |
| imag/irr | 2 |

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 1 & 0 & -20 \\ + & \downarrow & 2 & 4 & 10 & 20 \\ \hline & 1 & 2 & 5 & 10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 5 & 10 \\ + & \downarrow & -2 & 5 & -10 \\ \hline & 1 & 0 & 5 & 0 \end{array}$$

$x^2+5=0$

$x^2=-5$

$x=\pm\sqrt{-5} \rightarrow x=\pm i\sqrt{5}$

$$\boxed{\text{ROOTS: } x=2,-2,\pm i\sqrt{5}}$$