HM3 Polynomials Review

①

$$\begin{array}{r} 4x^2+10x+5 \\ 2x-1 \,\overline{\big)\, 8x^3+16x^2+0x-6} \\ 8x^3-4x^2 \\ \hline 20x^2+0x \\ 20x^2-10x \\ \hline 10x-6 \\ -\;10x-5 \\ \hline -1 \end{array}$$

$$\boxed{4x^2+10x+5-\frac{1}{2x-1}}$$

②

$$\begin{array}{r} 3x^2-2x+1 \\ x^2-x+4 \,\overline{\big)\, 3x^4-5x^3+15x^2-4x+3} \\ -\;3x^4-3x^3+12x^2 \\ \hline -2x^3+3x^2-4x \\ +2x^3+2x^2+8x \\ \hline x^2+4x+3 \\ -\;x^2-x+4 \\ \hline 5x-1 \end{array}$$

$$\boxed{3x^2-2x+1+\frac{5x-1}{x^2-x+4}}$$

③ ★must ÷ EVERYTHING by 3★

$(x^3-\frac{8}{3}x^2-\frac{35}{3}x+8)\div(x-\frac{2}{3})$

$$\begin{array}{r|rrrr} \frac{2}{3} & 1 & -\frac{8}{3} & -\frac{35}{3} & 8 \\ + & \downarrow & \frac{2}{3} & -\frac{4}{3} & -\frac{26}{3} \\ \hline & 1 & -2 & -13 & -\frac{2}{3} \end{array}$$

$x^2-2x-13-\dfrac{2/3}{x-2/3}$ ★can't leave a fraction in a fraction!

★multiply just the remainder by 3

$$\boxed{x^2-2x-13-\frac{2}{3x-2}}$$

④ ★must ÷ EVERYTHING by 3★

$(9x^3+8/3)\div(x+2/3)$

$$\begin{array}{r|rrrr} -\frac{2}{3} & 9 & 0 & 0 & \frac{8}{3} \\ + & \downarrow & -6 & 4 & -\frac{8}{3} \\ \hline & 9 & -6 & 4 & 0 \end{array}$$

$$\boxed{9x^2-6x+4}$$

④ $f(x) = 3x^4 + x^3 - 2x^2 + 4$

4 roots

$p = 4$ $q = 3$ possible rational:

1, 2, 4 | 1, 3 | $\pm 1, \frac{1}{3}, 2, \frac{2}{3}, 4, \frac{4}{3}$

$f(x) = P \rightarrow N \rightarrow P$ 2 or 0 +R

$f(-x) = 3(-x)^4 + (-x)^3 - 2(-x)^2 + 4$

$f(-x) = 3x^4 - x^3 - 2x^2 + 4$

$P \rightarrow N \rightarrow P$ 2 or 0 −R

+R	2	2	0	0
−R	2	0	2	0
imag/irr	0	2	2	4

⑤ $f(x) = x^3 + 8x^2 + 17x + 6$

3 roots

$p = 6$ $q = 1$

1, 2, 3, 6 | 1

possible rational: $\pm 1, 2, 3, 6$

$f(x) = P$ 0 +R

$f(-x) = (-x)^3 + 8(-x)^2 + 17(-x) + 6$

$f(-x) = -x^3 + 8x^2 - 17x + 6$

$N \rightarrow P \rightarrow N \rightarrow P$ 3 or 1 −R

+R	0	0
−R	3	1
imag/irr	0	2

⑥ $f(x) = x^3 - 12x + 16$

3 roots

$p = 16$ $q = 1$

1, 2, 4, 8, 16

possible rational: $\pm 1, 2, 4, 8, 16$

$f(x) = P \rightarrow N \rightarrow P$

2 or 0 +R

$f(-x) = (-x)^3 - 12(-x) + 16$

$f(-x) = -x^3 + 12x + 16$

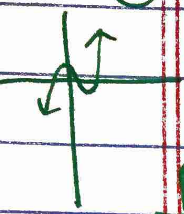
$N \rightarrow P$ 1 −R

+R	2	0
−R	1	1
imag/irr	0	2

Complete the chart:

function	possible rational roots	number of possible positive real roots	number of possible negative real roots	number of possible imaginary roots
4. $f(x) = 3x^4 + x^3 - 2x^2 + 4$	$\pm 1, \frac{1}{3}, 2, \frac{2}{3}, 4, 4/3$	2 or 0	2 or 0	0 or 2 or 4
5. $f(x) = x^3 + 8x^2 + 17x + 6$	$\pm 1, \frac{1}{3}, 2, \frac{2}{3}, 3, 6$	0	3 or 1	0 or 2
6. $f(x) = x^3 - 12x + 16$	$\pm 1, 2, 4, 8, 16$	2 or 0	1	0 or 2

⑦ $f(x) = 4x^3 - 4x^2 - x + 1$



3 roots

calc table: $(1, 0)$

1	4	-4	-1	1
+	↓	4	0	-1
	4	0	-1	0

$4x^2 - 1 = 0$

$4x^2 = 1$

$x^2 = 1/4$

$x = \pm\sqrt{1/4}$

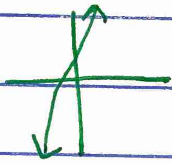
$x = \pm 1/2$

ROOTS: $x = 1, \pm 1/2$

★could have also used 2nd trace to find $x = \pm 1/2$★

⑧ $f(x) = 2x^3 + 5x^2 + 6x + 2$

3 roots



★nothing listed in calc. table so use 2nd trace zero★

$(-1/2, 0)$

-1/2	2	5	6	2
+	↓	-1	-2	-2
	2	4	4	0

$2x^2 + 4x + 4 = 0$ ★can ÷ by 2★

$x^2 + 2x + 2 = 0$

$x^2 + 2x + \underline{1} = -2 + \underline{1}$

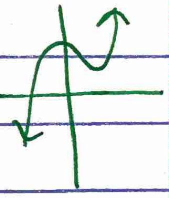
$(x+1)^2 = -1$

$x + 1 = \pm i$

$x = -1 \pm i$

ROOTS: $x = -1/2, -1 \pm i$

⑨ $f(x) = x^3 - 6x + 9$



calc table: $(-3, 0)$

-3	1	0	-6	9
+	↓	-3	9	-9
	1	-3	3	0

$x^2 - 3x + 3 = 0$

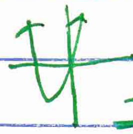
$x = \dfrac{3 \pm \sqrt{(-3)^2 - 4(1)(3)}}{2(1)}$

$x = \dfrac{3 \pm \sqrt{-3}}{2}$

$x = \dfrac{3 \pm i\sqrt{3}}{2}$

ROOTS: $x = -3, \dfrac{3 \pm i\sqrt{3}}{2}$

⑩ $f(x) = x^4 + 2x^3 + 5x^2 + 34x + 30$



calc table: $(-3, 0)$ $(-1, 0)$

-3	1	2	5	34	30
+	↓	-3	3	-24	-30
	1	-1	8	10	0

-1	1	-1	8	10
+	↓	-1	2	-10
	1	-2	10	0

$x^2 - 2x + 10 = 0$

$x^2 - 2x + \underline{1} = -10 + \underline{1}$

$(x-1)^2 = -9$

$x - 1 = \pm 3i$

$x = 1 \pm 3i$

ROOTS: $x = -3, -1, 1 \pm 3i$

(11) $f(x) = x^3 - 5x^2 + 3x - 2$

synthetic division: OR plug & chug:

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 3 & -2 \\ + & \downarrow & 1 & -4 & -1 \\ \hline & 1 & -4 & -1 & -3 \end{array}$$

NOT a factor

$f(1) = (1)^3 - 5(1)^2 + 3(1) - 2$

$f(1) = -3$

NOT a factor

(12) $x = 4 \quad x = -3 \quad x = -5$

$(x-4)(x+3)(x+5)$

$(x^2 - x - 12)(x+5)$

$x^3 + 5x^2 - x^2 - 5x - 12x - 60$

$f(x) = x^3 + 4x^2 - 17x - 60$

(13) $x = 8 \quad x = -5i$ ★ $x = 5i$ is also a root

$(x-8)$

sum: 0

prod: $-25i^2 \to 25$

$\downarrow$

$(x-8)(x^2+25)$

$x^3 + 25x - 8x^2 - 200$

$f(x) = x^3 - 8x^2 + 25x - 200$

(14) height $= x$

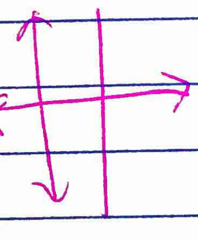
width $= x - 3$

length $= x + 3$

$80 = x(x-3)(x+3)$

$80 = (x^2 - 3x)(x+3)$

$80 = x^3 + 3x^2 - 3x^2 - 9x$

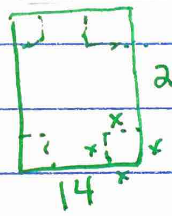


$0 = x^3 - 9x - 80$

★Graph in calc: $(5, 0)$

Dimensions: $5 \times 2 \times 8$

(15)



length: $20 - 2x$

width: $14 - 2x$

height: x

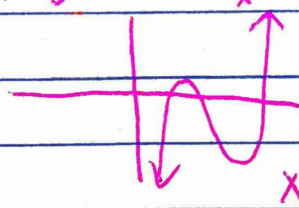
$336 = (20-2x)(14-2x)(x)$

$336 = (280 - 68x + 4x^2)x$

$4x^3 - 68x^2 + 280x - 336 = 0$

★can $\div$ by 4 before graphing★

$x^3 - 17x^2 + 70x - 84 = 0$

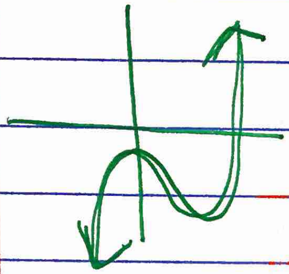


★make XMAX bigger to see all roots★

$x = 2.42, \quad x = 3, \quad \cancel{x = 11.58}$ too big!

★Find the LARGEST POSSIBLE size of the square: 3×3

(16) $f(x) = x^3 - 6x^2 + x - 4$



★ make YMIN smaller to see the whole graph

★ no absolute max or min

★ relative max at $x = .09$

★ relative min at $x = 3.91$

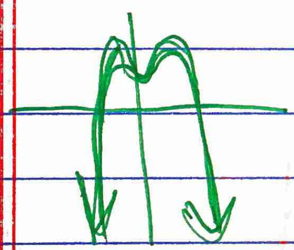
★ increasing: $(-\infty, .09)\ (3.91, \infty)$

★ decreasing: $(.09, 3.91)$

★ as $x \to -\infty$, $y \to -\infty$

★ as $x \to \infty$, $y \to \infty$

(17) $f(x) = -2x^4 + 6x^2 + 8$



★ make YMAX bigger to see the whole graph

★ absolute max at $x = -1.22$ and $x = 1.22$

★ relative min at $x = 0$

★ increasing: $(-\infty, -1.22)\ (0, 1.22)$

★ decreasing: $(-1.22, 0)\ (1.22, \infty)$

★ as $x \to -\infty$, $y \to -\infty$

★ as $x \to \infty$, $y \to -\infty$