

HM3

Name Key Fall 1b

Writing Equations Given Zeros

A polynomial function with rational coefficients has the following zeros. Find all additional zeros. DO NOT WRITE THE EQUATION OF THE FUNCTION.

1) $-3 + 2i, -3 + \sqrt{6}$

$-3-2i \quad -3-\sqrt{6}$

2) $\sqrt{6}, 1+i$

$-\sqrt{6} \quad 1-i$

3) $3 + \sqrt{10}, 3 + \sqrt{7}$

$3-\sqrt{10} \quad 3-\sqrt{7}$

4) $-2i, -2 + \sqrt{3}$

$2i \quad -2-\sqrt{3}$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

5) $\frac{2}{3}, 1+3i, 2i$

$f(x) = 3x^5 - 8x^4 + 46x^3 - 52x^2 + 136x - 80$

6) 3 mult. 2, 0, -5

$f(x) = x^4 - x^3 - 21x^2 + 45x$

7) 0 mult. 3

$f(x) = x^3$

8) 1, $2+\sqrt{7}$

$f(x) = x^3 - 5x^2 + x + 3$

9) -2 mult. 3

$f(x) = x^3 + 6x^2 + 12x + 8$

10) -1, $2-i$

$f(x) = x^3 - 3x^2 + x + 5$

11) 4, $-i$

$f(x) = x^3 - 4x^2 + x - 4$

12) -1 mult. 2, 2

$f(x) = x^3 - 3x - 2$

13) $\frac{4}{3}, -3i$

$f(x) = 3x^3 - 4x^2 + 27x - 36$

14) -4, 4, $-\frac{1}{4}$

$f(x) = 4x^3 + x^2 - 64x - 16$

$$(5) \quad x = 2/3, 1+3i, 2i$$

\downarrow
 $(x - 2/3)$
 \downarrow
 $(3x-2)$

$1-3i$ $\underline{\text{sum: } [2]}$	$-2i$ $\underline{\text{sum: } [0]}$
$\underline{\text{prod: } [1-9i^2]}$ $\underline{[10]}$	$\underline{\text{prod: } [4]}$

HW 2 WORKED OUT

$$(3x-2)(x^2-2x+10)(x^2+4)$$

$$(3x-2)(x^4 + 4x^2 - 2x^3 - 8x + 10x^2 + 40)$$

$$(3x-2)(x^4 - 2x^3 + 14x^2 - 8x + 40)$$

$$3x^5 - 6x^4 + 42x^3 - 24x^2 + 120x - 2x^4 + 4x^3 - 28x^2 + 16x - 80$$

$$f(x) = 3x^5 - 8x^4 + 46x^3 - 52x^2 + 136x - 80$$

⑥ 3 mult. 2, 0, -5
 $(x-3)(x-3) \times (x+5)$
 $x(x^2 - 6x + 9)(x+5)$
 $(x^3 - 6x^2 + 9x)(x+5)$
 $x^4 + 5x^3 - 6x^3 - 30x^2 + 9x^2 + 45x$

$f(x) = x^4 - x^3 - 21x^2 + 45x$

⑧ 1, $2+\sqrt{7}$
 $2-\sqrt{7}$
sum: 4
product: $4 - 7 = -3$

$(x-1)(x^2 - 4x - 3)$

$x^3 - 4x^2 - 3x - x^2 + 4x + 3$

$f(x) = x^3 - 5x^2 + x + 3$

⑩ -1, $2-i$ $\overset{\text{sum}}{2+i}$ $\overset{\text{prod}}{4-i^2}$

$(x+1)(x^2 - 4x + 5)$ ⑤

$x^3 - 4x^2 + 5x + x^2 - 4x + 5$

$f(x) = x^3 - 3x^2 + x + 5$

⑦ 0 mult. 3
 $(x)(x)(x)$

$y = x^3$

⑨ -2 mult. 3
 $(x+2)(x+2)(x+2)$
 $(x+2)(x^2 + 4x + 4)$
 $x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$

$f(x) = x^3 + 6x^2 + 12x + 8$

⑪ 4, $-i$ $\overset{\text{sum}}{i}$ $\overset{\text{prod}}{-i^2} = 1$

$(x-4)(x^2 + 1)$

$x^3 + x - 4x^2 - 4$

$f(x) = x^3 - 4x^2 + x - 4$

(12) -1 mult. 2, 2

$$(x+1)(x+1)(x-2)$$

$$(x^2+2x+1)(x-2)$$

$$x^3 - 2x^2 + 2x^2 - 4x + x - 2$$

$$f(x) = x^3 - 3x - 2$$

(13) $\frac{4}{3}, -3i, 3i$

$$(3x-4)$$

sum: 0
prod: $-9i^2 \rightarrow 9$

$$(3x-4)(x^2+9)$$

$$3x^3 + 27x - 4x^2 - 36$$

$$f(x) = 3x^3 - 4x^2 + 27x - 36$$

(14) $-4, 4, -\frac{1}{4}$

$$(x+4)(x-4)(4x+1)$$

$$(x^2 - 16)(4x+1)$$

$$4x^3 + x^2 - 64x - 16$$

$$f(x) = 4x^3 + x^2 - 64x - 16$$