

Honors Math 3 - Complex Rational Expressions

$$\textcircled{1} \frac{\frac{4h}{9r^3}}{\frac{8h^4}{15r^2}} = \frac{\frac{4h}{9r^3} \cdot \frac{15r^2}{8h^4}}{1} = \frac{5}{6rn^3}$$

$$\textcircled{2} \frac{a \cdot \frac{1}{a} - a \cdot a}{\frac{3}{a} \cdot a} = \frac{1 - a^2}{3}$$

CD: a

$$\textcircled{3} \frac{x \cdot \frac{7}{x} - b \cdot x}{x \cdot \frac{2}{x} + 3 \cdot x} = \frac{7 - bx}{2 + 3x}$$

CD: x

$$\textcircled{4} \frac{\frac{2}{7x} - \frac{3}{14} \cdot 14x}{\frac{3}{14x} - \frac{1}{7} \cdot 14x} = \frac{4 - 3x}{3 - 2x}$$

CD: 14x

$$\textcircled{5} \frac{a^2 + \frac{2a}{3}}{\frac{a^2}{6} + 14a} = \frac{6a^2 + 4a}{a^2 + 84a}$$

CD: 6

$$\frac{2a(3a+2)}{a(a+84)}$$

$$\textcircled{6} \frac{\frac{2}{2c} + \frac{1}{2c} \cdot 2c}{2c \cdot c + \frac{c}{2} \cdot 2c} = \frac{4+1}{2c^2+c^2} = \frac{5}{3c^2}$$

CD: 2c

$$\frac{2(3a+2)}{a+84} \text{ or } \frac{6a+4}{a+84}$$

$$\begin{aligned} &10 \cdot \frac{2x}{5} + \frac{3y}{10} \cdot 10 \\ &10 \cdot \frac{3x}{2} - \frac{7y}{10} \cdot 10 \end{aligned}$$

$$\frac{4x+3y}{15x-7y}$$

CD: 10

$$\begin{aligned} &8 \cdot \frac{c}{3cd} - \frac{d}{3c} \cdot 3cd \\ &3cd \cdot \frac{1}{3c} + \frac{1}{3d} \cdot 3cd \end{aligned}$$

$$\frac{c^2-d^2}{d+c}$$

*factor!

$$\frac{(c-d)(c+d)}{c+d} = c-d$$

CD: 3cd

$$\begin{aligned} &9 \cdot 1 + \frac{1}{2a} \cdot 4a \\ &4a \cdot \frac{1}{4a} - a \cdot 4a \end{aligned}$$

$$\frac{4a+2}{1-4a^2}$$

*factor!

$$\frac{2(2a+1)}{(1-2a)(1+2a)}$$

$$\frac{2}{1-2a}$$

CD: 4a

$$\begin{aligned} &10 \cdot (y+4) + \frac{4}{y} \cdot y \\ &y \cdot (y+1) - \frac{2}{y} \cdot y \end{aligned}$$

$$\frac{y^2+4y+4}{y^2+y-2}$$

*factor!

$$\frac{(y+2)(y+2)}{(y+2)(y-1)} = \frac{y+2}{y-1}$$

CD: y

$$\begin{aligned} &11 \cdot 9u^4 - \frac{1}{w^2} \\ &3u^2 + \frac{1}{w} \end{aligned}$$

$$\frac{9u^4w^2-1}{3u^2w^2+w}$$

*factor!

• top is diff. of perf. □s

• bottom has a GCF

$$\frac{(3u^2w-1)(3u^2w+1)}{w(3u^2w+1)}$$

$$\frac{3u^2w-1}{w}$$

CD: w²

$$\begin{aligned} &12 \cdot \frac{w}{w-z} - 1 \cdot w-z \\ &w-z \cdot w-z \end{aligned}$$

$$\frac{w-(w-z)}{w}$$

$$\frac{w-w+z}{w} = \frac{z}{w}$$

CD: w-z

$$\frac{5}{x+y} + 3(x+y)$$

$$\frac{3}{x+y} + 7(x+y)$$

CD: $x+y$

$$\frac{5+3x+3y}{3+7x+7y}$$

$$\frac{u-2 - \frac{4}{u-2}}{u-1 - \frac{2}{u-2}} = \frac{(u-2)(u-2) - 4}{(u-1)(u-2) - 2}$$

CD: $u-2$

$$\frac{u^2 - 4u + 4 - 4}{u^2 - 3u + 2 - 2}$$

$$\frac{u^2 - 4u}{u^2 - 3u} \xrightarrow{\text{factor}} \frac{u(u-4)}{u(u-3)}$$

$$\frac{u-4}{u-3}$$

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$$\frac{\frac{n}{n-2} - \frac{2}{n+2} - \frac{8}{n^2-4}}{\frac{1}{n+2}}$$

* MUST factor $(n-2)(n+2)$

CD: $(n-2)(n+2)$

$$\frac{n(n+2) - 2(n-2) - 8}{n+2}$$

$$\frac{n^2 + 2n - 2n + 4 - 8}{n+2} = \frac{n^2 - 4}{n+2} = \frac{(n-2)(n+2)}{n+2}$$

$$n-2$$

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$$\frac{3a}{a^2-9} \xrightarrow{\text{factor}} \frac{3a}{(a-3)(a+3)}$$

$$\frac{1}{3a} - \frac{1}{3+a}$$

factor out a

$$-1: \frac{1}{-1(-3+a)} \rightarrow \frac{-1}{a-3}$$

CD: $(a+3)(a-3)$

$$\frac{3a}{-1(a+3) - 1(a-3)}$$

$$\frac{3a}{-a-3-a+3}$$

$$\frac{3a}{-2a}$$

$$\frac{-3}{2}$$

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$$\frac{9 + \frac{6}{c} + \frac{1}{c^2}}{\frac{3}{c} + \frac{1}{c^2}}$$

CD: c^2

$$\frac{9c^2 + 6c + 1}{3c + 1}$$

* factor top!

$$\frac{(3c+1)(3c+1)}{3c+1}$$

$$3c+1$$

$$\frac{1}{p+1} - 1$$

$$\frac{1}{p^2-1} + 1$$

Must factor $(p+1)(p-1)$

$$\frac{p-1 - (p+1)(p-1)}{1 + (p+1)(p-1)}$$

$$\frac{p-1 - (p^2-1)}{1 + p^2-1}$$

CD: $(p+1)(p-1)$

$$\frac{p-1-p^2+1}{p^2}$$

$$\frac{p(1-p)}{p^2} \quad \frac{1-p}{p}$$

or $\frac{-p+1}{p}$

*this is one of the "long problems"

19) $2 - \frac{1}{2 + \frac{1}{2 - \frac{1}{a}}}$

start here

CD: a $\frac{2a-1}{a} - \frac{1}{a} \rightarrow \frac{2a-1}{a}$

$$2 - \frac{1}{2 + \frac{1}{\frac{2a-1}{a}}} \rightarrow 1 \div \frac{2a-1}{a} \rightarrow 1 \cdot \frac{a}{2a-1} = \frac{a}{2a-1}$$

keep, change, flip!

$$2 - \frac{1}{2 + \frac{a}{2a-1}} \quad \text{CD: } 2a-1 \quad \frac{2(2a-1)}{2a-1} + \frac{a}{2a-1} = \frac{4a-2+a}{2a-1} = \frac{5a-2}{2a-1}$$

$$2 - \frac{1}{\frac{5a-2}{2a-1}} \rightarrow 1 \div \frac{5a-2}{2a-1} \rightarrow 1 \cdot \frac{2a-1}{5a-2} = \frac{2a-1}{5a-2}$$

keep, change, flip!

$$2 - \frac{2a-1}{5a-2} \quad \text{CD: } 5a-2 \quad \frac{2(5a-2)}{5a-2} - \frac{2a-1}{5a-2}$$

$$\frac{10a-4-2a+1}{5a-2} \rightarrow \frac{8a-3}{5a-2}$$

$$2 + \frac{1}{2+x} \quad \text{CD: } 2+x \quad \frac{2(2+x)}{2+x} + \frac{1}{2+x} = \frac{4+2x+1}{2+x} = \frac{2x+5}{2+x}$$

$$2 + \frac{\frac{1}{2x+5}}{2+x} \quad \text{keep, change, flip} \quad 1 \div \frac{2x+5}{2+x} \rightarrow 1 \cdot \frac{2+x}{2x+5} = \frac{2+x}{2x+5}$$

$$2 + \frac{2+x}{2x+5} \quad \text{CD: } 2x+5 \quad \frac{2(2x+5)}{2x+5} + \frac{2+x}{2x+5} = \frac{4x+10+2+x}{2x+5} = \frac{5x+12}{2x+5}$$

$$21 \quad 1 - \frac{1}{2 - \frac{1}{3-r}} \quad \text{CD: } 3-r \quad \frac{2(3-r)}{3-r} - \frac{1}{3-r} = \frac{6-2r-1}{3-r} = \frac{5-2r}{3-r}$$

$$1 - \frac{\frac{1}{5-2r}}{3-r} \quad \text{keep, change, flip} \quad 1 \div \frac{5-2r}{3-r} \rightarrow 1 \cdot \frac{3-r}{5-2r} = \frac{3-r}{5-2r}$$

$$1 - \frac{3-r}{5-2r} \quad \text{CD: } 5-2r \quad \frac{5-2r}{5-2r} - \frac{3-r}{5-2r} \rightarrow \frac{5-2r-3+r}{5-2r} = \frac{2-r}{5-2r}$$

$$\frac{1}{2-r} - \frac{2-r}{5-2r} \quad \text{keep, change, flip} \quad 1 \div \frac{2-r}{5-2r} \rightarrow 1 \cdot \frac{5-2r}{2-r} = \frac{5-2r}{2-r}$$

$$\textcircled{22} \frac{6}{a-2} + \frac{5}{a+2}$$

$$\frac{6(a+2) + 5(a-2)}{7 + 2(a+2)}$$

$$\rightarrow \frac{7}{a^2-4} + \frac{2}{a-2}$$

★ factor!
 $(a-2)(a+2)$ CD: $(a-2)(a+2)$

$$\frac{6a+12+5a-10}{7+2a+4}$$

$$\boxed{\frac{11a+2}{2a+11}}$$

$$\textcircled{23} \frac{y^2+8y+15}{y^2+y-6} \cdot \frac{y^2+2y-15}{y^2-2y-3}$$

★ factor everything! ★ keep, change, flip!

$$\frac{(y+5)(y+3)}{(y+3)(y-2)} \cdot \frac{(y-3)(y+1)}{(y+5)(y-3)} = \boxed{\frac{y+1}{y-2}}$$

$$\textcircled{24} \frac{p+4}{p-6} + \frac{p+1}{p+2}$$

$$\frac{(p+4)(p+2) + (p+1)(p-6)}{2p^2+3}$$

★ factor!
 $(p-6)(p+2)$ CD: $(p-6)(p+2)$

$$\frac{p^2+6p+8 + p^2-5p-6}{2p^2+3}$$

$$\boxed{\frac{2p^2+p+2}{2p^2+3}}$$

$$\textcircled{1} \quad x-2 + \frac{9x+11}{x+3} \quad \underline{\text{CD}}: (x+3)$$

$$\frac{x+5 + \frac{x-15}{x+4}}{\underline{\text{CD}}: x+4}$$

this works best if you find an individual CD for numerator & denom.

$$\frac{x^2 + x - 6 + 9x + 11}{x+3} \div \frac{x^2 + 9x + 20 + x - 15}{x+4}$$

$$\frac{x^2 + 10x + 5}{x+3} \cdot \frac{x+4}{x^2 + 10x + 5}$$

$$\boxed{\frac{x+4}{x+3}}$$

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$$\frac{u-4}{u-1 - \frac{15}{u+1}} \quad \underline{\text{CD}}: u+1$$

$$\frac{u-4}{\frac{u^2-1-15}{u+1}}$$

$$\frac{u-4}{\frac{u^2-16}{u+1}}$$

$$\frac{u-4}{1} \div \frac{(u-4)(u+4)}{u+1}$$

$$\frac{\cancel{u-4}}{1} \cdot \frac{u+1}{\cancel{(u-4)}(u+4)}$$

$$\boxed{\frac{u+1}{u+4}}$$