

#M3 - Verifying Trig Identities

$$\textcircled{1} \sin x \csc x = 1$$

$$\sin x \cdot \frac{1}{\sin x} = 1$$

$$1 = 1 \checkmark$$

$$\textcircled{2} \tan \theta \cot \theta = 1$$

$$\tan \theta \cdot \frac{1}{\tan \theta} = 1$$

$$1 = 1 \checkmark$$

$$\textcircled{3} (1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta$$

FOIL!

$$1 - \sin \theta + \sin \theta - \sin^2 \theta = \cos^2 \theta$$

$$1 - \sin^2 \theta = \cos^2 \theta$$

$$\cos^2 \theta = \cos^2 \theta \checkmark$$

$$\textcircled{4} \cot^2 \theta (\sec^2 \theta - 1) = 1$$

Pythag ID

$$\cot^2 \theta (\tan^2 \theta) = 1$$

$$\frac{1}{\tan^2 \theta} \cdot \frac{\tan^2 \theta}{1} = 1$$

$$1 = 1 \checkmark$$

$$\textcircled{5} \cos^2 \theta - \sin^2 \theta = 1 - 2\sin^2 \theta$$

change to Pythag ID

$$1 - \sin^2 \theta - \sin^2 \theta = 1 - 2\sin^2 \theta$$

$$1 - 2\sin^2 \theta = 1 - 2\sin^2 \theta \checkmark$$

$$\textcircled{6} \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$$

change to Pythag ID

$$\cos^2 \theta - (1 - \cos^2 \theta) = 2\cos^2 \theta - 1$$

$$\cos^2 \theta - 1 + \cos^2 \theta = 2\cos^2 \theta - 1$$

$$2\cos^2 \theta - 1 = 2\cos^2 \theta - 1 \checkmark$$

$$\textcircled{7} \tan^2 \theta + 4 = \sec^2 \theta + 3$$

Pythag ID

$$\sec^2 \theta - 1 + 4 = \sec^2 \theta + 3$$

$$\sec^2 \theta + 3 = \sec^2 \theta + 3 \checkmark$$

$$\textcircled{8} 2 - \sec^2 \theta = 1 - \tan^2 \theta$$

Pythag ID

$$2 - (1 + \tan^2 \theta) = 1 - \tan^2 \theta$$

$$2 - 1 - \tan^2 \theta = 1 - \tan^2 \theta$$

$$1 - \tan^2 \theta = 1 - \tan^2 \theta \checkmark$$

$$\textcircled{9} \sin^2 \theta - \sin^4 \theta = \cos^2 \theta - \cos^4 \theta$$

FACTOR out GCF

$$\sin^2 \theta (1 - \sin^2 \theta) = \cos^2 \theta - \cos^4 \theta$$

*use Pythag. ID's for both

$$(1 - \cos^2 \theta)(\cos^2 \theta) = \cos^2 \theta - \cos^4 \theta$$

$$\cos^2 \theta - \cos^4 \theta = \cos^2 \theta - \cos^4 \theta \checkmark$$

$$\cos \theta + \sin \theta + \tan \theta = \sec \theta$$

$$\cos \theta + \sin \theta \cdot \frac{\sin \theta}{\cos \theta} = \sec \theta$$

$$\cos \theta + \frac{\sin^2 \theta}{\cos \theta} = \sec \theta$$

get a common denominator

$$\frac{\cos^2 \theta}{\cos \theta} + \frac{\sin^2 \theta}{\cos \theta} = \sec \theta$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} = \sec \theta$$

$$\frac{1}{\cos \theta} = \sec \theta$$

$$\sec \theta = \sec \theta \checkmark$$

$$(12) \frac{\sin \theta}{1 - \cos \theta} = \frac{1 + \cos \theta}{\sin \theta}$$

multiply by the conjugate

$$\frac{\sin \theta}{1 - \cos \theta} \cdot \frac{1 + \cos \theta}{1 + \cos \theta} = \frac{1 + \cos \theta}{\sin \theta}$$

$$\frac{\sin \theta (1 + \cos \theta)}{1 - \cos^2 \theta} = \frac{1 + \cos \theta}{\sin \theta}$$

$$\frac{\sin \theta (1 + \cos \theta)}{\sin^2 \theta} = \frac{1 + \cos \theta}{\sin \theta}$$

$$\frac{1 + \cos \theta}{\sin \theta} = \frac{1 + \cos \theta}{\sin \theta} \checkmark$$

$$(11) \frac{\sec^2 \theta}{\tan \theta} = \sec \theta \csc \theta$$

$$\sec \theta \cdot \frac{\sec \theta}{\tan \theta} = \sec \theta \csc \theta$$

$$\sec \theta \cdot \frac{\frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} = \sec \theta \csc \theta$$

$$\sec \theta \left(\frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} \right) = \sec \theta \csc \theta$$

$$\sec \theta \cdot \frac{1}{\sin \theta} = \sec \theta \csc \theta$$

$$\sec \theta \csc \theta = \sec \theta \csc \theta$$

$$(13) \frac{1}{\sin \theta} - \sin \theta = \frac{\cos^2 \theta}{\sin \theta} \checkmark$$

get a common denominator

$$\frac{1}{\sin \theta} - \frac{\sin^2 \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta}$$

$$\frac{1 - \sin^2 \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta}$$

$$\frac{\cos^2 \theta}{\sin \theta} = \frac{\cos^2 \theta}{\sin \theta} \checkmark$$

$$\sin^{\frac{1}{2}} \theta \cos \theta - \sin^{\frac{5}{2}} \theta \cos \theta = \cos^3 \theta \sqrt{\sin \theta}$$

Factor out GCF: $\sin^{\frac{1}{2}} \theta \cos \theta$

$$\sin^{\frac{1}{2}} \theta \cos \theta (1 - \sin^2 \theta) = \cos^3 \theta \sqrt{\sin \theta}$$

$$\sin^{\frac{1}{2}} \theta \cos \theta (1 - \sin^2 \theta) = \cos^3 \theta \sqrt{\sin \theta}$$

$$\sin^{\frac{1}{2}} \theta \cos \theta (\cos^2 \theta) = \cos^3 \theta \sqrt{\sin \theta}$$

$$\sqrt{\sin \theta} \cos^3 \theta = \cos^3 \theta \sqrt{\sin \theta}$$

$$\cos^3 \theta \sqrt{\sin \theta} = \cos^3 \theta \sqrt{\sin \theta} \checkmark$$

$$(15) \frac{1}{\sec \theta \tan \theta} = \csc \theta - \sin \theta$$

$$\frac{1}{\sec \theta} \cdot \frac{1}{\tan \theta} = \csc \theta - \sin \theta$$

$$\cos \theta \cdot \cot \theta = \csc \theta - \sin \theta$$

$$\cos \theta \cdot \frac{\cos \theta}{\sin \theta} = \csc \theta - \sin \theta$$

$$\frac{\cos^2 \theta}{\sin \theta} = \csc \theta - \sin \theta$$

$$\frac{1 - \sin^2 \theta}{\sin \theta} = \csc \theta - \sin \theta$$

$$\frac{1}{\sin \theta} - \frac{\sin^2 \theta}{\sin \theta} = \csc \theta - \sin \theta$$

$$\csc \theta - \sin \theta = \csc \theta - \sin \theta \checkmark$$

$$(16) \csc^4 \theta - 2 \csc^2 \theta + 1 = \cot^4 \theta$$

Factor!!
similar to $x^4 - 2x^2 + 1$

$$(\csc^2 \theta - 1)(\csc^2 \theta - 1) = \cot^4 \theta$$

$$\frac{(\csc^2 \theta - 1)^2}{\text{Pythag ID}} = \cot^4 \theta$$

$$(\cot^2 \theta)^2 = \cot^4 \theta$$

$$\cot^4 \theta = \cot^4 \theta \checkmark$$

$$(17) \csc \theta - \sin \theta = \cos \theta \cot \theta$$

$$\frac{1}{\sin \theta} - \sin \theta = \cos \theta \cot \theta$$

get a common denominator

$$\frac{1}{\sin \theta} - \frac{\sin^2 \theta}{\sin \theta} = \cos \theta \cot \theta$$

$$\frac{1 - \sin^2 \theta}{\sin \theta} = \cos \theta \cot \theta$$

$$\frac{\cos^2 \theta}{\sin \theta} = \cos \theta \cot \theta$$

$$\cos \theta \cdot \frac{\cos \theta}{\sin \theta} = \cos \theta \cot \theta$$

$$\cos \theta \cot \theta = \cos \theta \cot \theta \checkmark$$

$$\sec \theta - \cos \theta = \sin \theta \tan \theta$$

$$\frac{1}{\cos \theta} - \cos \theta = \sin \theta \tan \theta$$

get a common denominator

$$\frac{1}{\cos \theta} - \frac{\cos^2 \theta}{\cos \theta} = \sin \theta \tan \theta$$

$$\frac{1 - \cos^2 \theta}{\cos \theta} = \sin \theta \tan \theta$$

$$\frac{\sin^2 \theta}{\cos \theta} = \sin \theta \tan \theta$$

$$\sin \theta \cdot \frac{\sin \theta}{\cos \theta} = \sin \theta \tan \theta$$

$$\sin \theta \tan \theta = \sin \theta \tan \theta \checkmark$$

$$\textcircled{20} \frac{1}{\tan \theta} + \frac{1}{\cot \theta} = \tan \theta + \cot \theta$$

$$\cot \theta + \tan \theta = \tan \theta + \cot \theta$$

$$\tan \theta + \cot \theta = \tan \theta + \cot \theta \checkmark$$

$$\textcircled{22} \frac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$$

$$\frac{\cancel{\tan x} + \cancel{\cot y}}{\cancel{\tan x} \cancel{\cot y}} = \tan y + \cot x$$

$$\frac{1}{\cot y} + \frac{1}{\tan x} = \tan y + \cot x$$

$$\tan y + \cot x = \tan y + \cot x \checkmark$$

$$\textcircled{19} \cos \theta + \sin \theta \tan \theta = \sec \theta$$

$$\cos \theta + \sin \theta \cdot \frac{\sin \theta}{\cos \theta} = \sec \theta$$

$$\cos \theta + \frac{\sin^2 \theta}{\cos \theta} = \sec \theta$$

get a common denominator

$$\frac{\cos^2 \theta}{\cos \theta} + \frac{\sin^2 \theta}{\cos \theta} = \sec \theta$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} = \sec \theta$$

$$\frac{1}{\cos \theta} = \sec \theta$$

$$\sec \theta = \sec \theta \checkmark$$

$$\textcircled{21} \frac{1}{\sin \theta} - \frac{1}{\csc \theta} = \csc \theta - \sin \theta$$

$$\csc \theta - \sin \theta = \csc \theta - \sin \theta \checkmark$$

$$\frac{1}{\tan \theta + 1} + \frac{1}{\cot \theta + 1} = 1 \quad \star \text{get a common denominator} \star$$

$$\frac{\cot \theta + 1}{(\tan \theta + 1)(\cot \theta + 1)} + \frac{\tan \theta + 1}{(\tan \theta + 1)(\cot \theta + 1)} = 1$$

$$\frac{\cot \theta + \tan \theta + 2}{\tan \theta \cot \theta + \tan \theta + \cot \theta + 1} = 1 \quad \star \text{FOIL the denominator}$$

$$\frac{\cot \theta + \tan \theta + 2}{\tan \theta \cdot \frac{1}{\tan \theta} + \tan \theta + \cot \theta + 1} = 1$$

$$\frac{\cot \theta + \tan \theta + 2}{1 + \tan \theta + \cot \theta + 1} = 1$$

$$\frac{\cot \theta + \tan \theta + 2}{\cot \theta + \tan \theta + 2} = 1$$

$$1 = 1 \quad \checkmark$$

$$(24) \cos x - \csc x \cot x = -\cos x \cot^2 x$$

$$\cos x - \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\cos x \cot^2 x$$

$$\cos x - \frac{\cos x}{\sin^2 x} = -\cos x \cot^2 x$$

\star can factor out GCF: $\cos x$

$$\cos x \left(1 - \frac{1}{\sin^2 x} \right) = -\cos x \cot^2 x$$

$$\cos x \left(\frac{1 - \csc^2 x}{\text{Pythag ID}} \right) = -\cos x \cot^2 x$$

$$\cos x (-\cot^2 x) = -\cos x \cot^2 x$$

$$-\cos x \cot^2 x = -\cos x \cot^2 x$$

\checkmark