Math 3 Name Key Spring 17

# Sequences Extra Practice - MUST SHOW ALL WORK FOR CREDIT

**For each sequence, state if it is arithmetic, geometric, or neither.**

1) −10, −8, −5, −1, 4, ...

neither

2) −6, −10, −14, −18, −22, ...

Arithmetic $d=-4$

3) 4, −12, 36, −108, 324, ...

geometric $r=-3$

4) 2, $\frac{1}{2}$, $\frac{8}{27}$, $\frac{1}{4}$, $\frac{32}{125}$, ...

neither

5) 1, −5, 25, −125, 625, ...

geometric $r=-5$

6) 26, 22, 18, 14, 10, ...

arithmetic $d=-4$

**Find the explicit formula and the recursive formula.**

7) −32, −39, −46, −53, ... Arithmetic $d=-7$

Rec: $a_1=-32$ $a_n=a_{n-1}-7$

Exp: $a_n=-32+(n-1)(-7)$
$a_n=-32-7n+7$
$a_n=-25-7n$

*must distribute & combine like terms

8) −37, −30, −23, −16, ... Arithmetic $d=7$

Rec: $a_1=-37$ $a_n=a_{n-1}+7$

Exp: $a_n=-37+(n-1)(7)$
$a_n=-37+7n-7$
$a_n=7n-44$

9) −30, −130, −230, −330 Arithmetic $d=-100$

Rec: $a_1=-30$ $a_n=a_{n-1}-100$

Exp: $a_n=-30+(n-1)(-100)$
$a_n=-30-100n+100$
$a_n=70-100n$

10) −9, −7, −5, −3, ... Arithmetic $d=+2$

Rec: $a_1=-9$ $a_n=a_{n-1}+2$

Exp: $a_n=-9+(n-1)(2)$
$a_n=-9+2n-2$
$a_n=2n-11$

11) −3, 9, −27, 81, ... Geometric $r=-3$

Rec: $a_1=-3$ $a_n=a_{n-1}\cdot-3$

Exp: $a_n=-3(-3)^{n-1}$

12) −3, 6, −12, 24, ... Geometric $r=-2$

Rec: $a_1=-3$ $a_n=a_{n-1}\cdot-2$

Exp: $a_n=-3(-2)^{n-1}$

13) −4, 20, −100, 500, ... Geometric $r=-5$

Rec: $a_1=-4$ $a_n=a_{n-1}\cdot-5$

Exp: $a_n=-4(-5)^{n-1}$

14) −3, 18, −108, 648, ... Geometric $r=-6$

Rec: $a_1=-3$ $a_n=a_{n-1}\cdot-6$

Exp: $a_n=-3(-6)^{n-1}$

**Given the first term and the common difference of an arithmetic sequence list the next three terms.**

15) $a_1 = 40,\ d = 7$

40, 47, 54, 61

**Given the first term and the common ratio of a geometric sequence list the next three terms.**

16) $a_1 = 3,\ r = -5$

3, -15, 75, -375

**Find the term named in the problem.**

17) $-12, -7, -2, 3, \ldots$ Arithmetic $d=5$

Find $a_{34}$

$a_{34} = -12 + (34-1)(5)$

$a_{34} = 153$

the 34th term is 153

18) $18, -82, -182, -282, \ldots$ Arithmetic $d=-100$

Find $a_{21}$

$a_{21} = 18 + (21-1)(-100)$

$a_{21} = -1982$

19) $4, -2, 1, -\frac{1}{2}, \ldots$ Geometric $r=-1/2$

Find $a_{12}$

$a_{12} = 4(-1/2)^{12-1}$

$a_{12} = \frac{-1}{512}$

20) $-2, -10, -50, -250, \ldots$ Geometric $r=5$

Find $a_9$

$a_9 = -2(5)^{9-1}$

$a_9 = -781250$

**Given a term in an arithmetic sequence and the common difference find the term named in the problem.** *Manipulate the formula since you don't know $a_1$!*

21) $a_{23} = -206,\ d = -10$

Find $a_{33}$

$a_n = -206 + (n-23)(-10)$

$a_{33} = -206 + (33-23)(-10)$

$a_{33} = -306$

22) $a_{33} = 661,\ d = 20$

Find $a_{37}$

$a_n = 661 + (n-33)(20)$

$a_{37} = 661 + (37-33)(20)$

$a_{37} = 741$

**Given a term in a geometric sequence and the common ratio find the term named in the problem.** *Manipulate the formula since you don't know $a_1$!*

23) $a_6 = \frac{1}{8},\ r = \frac{1}{2}$

Find $a_{12}$

$a_n = \frac{1}{8}\left(\frac{1}{2}\right)^{n-6}$

$a_{12} = \frac{1}{8}\left(\frac{1}{2}\right)^{12-6}$

$a_{12} = \frac{1}{512}$

24) $a_6 = 6250,\ r = -5$

Find $a_9$

$a_n = 6250(-5)^{n-6}$

$a_9 = 6250(-5)^{9-6}$

$a_9 = -781250$

Arithmetic $d=2$

25. Which term is 8192 in the sequence

2, 4, 6, 8, ...

2
2
8192
?

$8192 = 2 + (n-1)(2)$

$8190 = (n-1)(2)$

$4096 = n-1$

$\boxed{4097 = n}$

Arithmetic $d=4$

26. How many terms are in the sequence

-11, -7, -3, ... 33

$a_1 = -11$
$d = 4$
$a_n = 33$
$n = ?$

$33 = -11 + (n-1)(4)$

$44 = (n-1)(4)$

$11 = n-1$

$\boxed{12 = n}$

27. Determine the number of terms in the sequence

-6, -10, -14, ... -74 Arithmetic $d=-4$

$a_1 = -6$
$d = -4$
$a_n = -74$
$n = ?$

$-74 = -6 + (n-1)(-4)$

$-68 = (n-1)(-4)$

$17 = n-1$

$\boxed{18 = n}$

28. Which term is 3125 in the sequence

$\frac{1}{625}, \frac{1}{625}, \frac{1}{125}, \frac{1}{25}, \ldots$ Geometric $r=5$

$a_1 = \frac{1}{625}$
$r = 5$
$a_n = 3125$
$n = ?$

$3125 = \frac{1}{625}(5)^{n-1}$

$1953125 = 5^{n-1}$

$\log_5 1953125 = n-1$

$9 = n-1$

$\boxed{10 = n}$

*Given the following two terms in an ARITHMETIC SEQUENCE, find the term named in the problem.*

29. $a_{11} = 226$ *and* $a_{35} = 706$. *Find* $a_{24}$.

manipulate eqn: $a_n = 226 + (n-11)d$

find d: $706 = 226 + (35-11)d$

$480 = 24d$

$20 = d$

update eqn: $a_n = 226 + (n-11)(20)$

find $a_{24}$: $a_{24} = 226 + (24-11)(20)$

$\boxed{a_{24} = 486}$

30. $a_{19} = -12$ *and* $a_{32} = -38$. *Find* $a_{37}$.

manipulate eqn: $a_n = -12 + (n-19)d$

find d: $-38 = -12 + (32-19)d$

$-26 = 13d$

$d = -2$

update eqn: $a_n = -12 + (n-19)(-2)$

find $a_{37}$: $a_{37} = -12 + (37-19)(-2)$

$\boxed{a_{37} = -48}$

*Given the following two terms in a GEOMETRIC SEQUENCE, find the term named in the problem.*

31. $a_2 = 12$ *and* $a_5 = -324$. *Find* $a_{10}$.

manipulate eqn: $a_n = 12(r)^{n-2}$

find r: $-324 = 12(r)^{5-2}$

$-27 = r^3$

$-3 = r$

update eqn: $a_n = 12(-3)^{n-2}$

find $a_{10}$: $a_{10} = 12(-3)^{10-2}$

$\boxed{a_{10} = 78732}$

32. $a_3 = 36$ *and* $a_6 = -972$. *Find* $a_{12}$.

manipulate eqn: $a_n = 36(r)^{n-3}$

find r: $-972 = 36(r)^{6-3}$

$-27 = r^3$

$-3 = r$

update eqn: $a_n = 36(-3)^{n-3}$

find $a_{12}$: $a_{12} = 36(-3)^{12-3}$

$\boxed{a_{12} = -708588}$

33. $a_3 = \frac{-2}{3}$ and $a_6 = \frac{16}{81}$. Find $a_{11}$.

anipulate eqn: $a_n = \frac{-2}{3}(r)^{n-3}$

ind r: $\frac{16}{81} = -\frac{2}{3}(r)^{6-3}$

$\frac{-8}{27} = r^3$

$r = -2/3$

pdate eqn: $a_n = \frac{-2}{3}\left(-\frac{2}{3}\right)^{n-3}$

find $a_{11}$: $a_{11} = \frac{-2}{3}\left(-\frac{2}{3}\right)^{11-3}$

$a_{11} = \frac{-512}{19683}$ or

$a_{11} = -.0260123$

**CHALLENGE PROBLEMS!!!** *Think you've mastered sequences? Give these more difficult problems a try!*

34. In an arithmetic sequence, $a_2 = 5k + 3j$ and $a_3 = 4k + 4j$. Find the 8th term.

anipulate eqn: $a_n = 5k+3j+(n-2)d$

find d: $4k+4j = 5k+3j+(3-2)d$

$4k+4j = 5k+3j+d$

$-k+j = d$

update eqn: $a_n = 5k+3j+(n-2)(-k+j)$

find $a_8$: $a_8 = 5k+3j+(8-2)(-k+j)$

$a_8 = 5k+3j+6(-k+j)$

$a_8 = 5k+3j-6k+6j$

$a_8 = -k+9j$

35. Find the 23rd term of the sequence: $5\sqrt{2}, -3\sqrt{2}, -11\sqrt{2}, \ldots$

Arithmetic $\rightarrow d = -8\sqrt{2}$

$a_{23} = 5\sqrt{2}+(23-1)(-8\sqrt{2})$

$a_{23} = 5\sqrt{2}+22(-8\sqrt{2})$

$a_{23} = 5\sqrt{2}-176\sqrt{2}$

$a_{23} = -171\sqrt{2}$

36. How many multiples of 13 are there between 33 and 215?

*multiples of 13 are #s that 13 will go into*

between 33 & 215:

smallest $\rightarrow 39$

largest $\rightarrow 208$

$a_1 = 39$

$a_n = 208$

$d = 13$

$n = ?$

$208 = 39+(n-1)(13)$

$169 = (n-1)(13)$

$13 = n-1$

$n = 14$

37. Find the 10th term (no decimals!): $-2, -2\sqrt{5}, -10, \ldots$

Geometric $\rightarrow r = \sqrt{5}$

$a_{10} = -2(\sqrt{5})^{10-1}$

$a_{10} = -2(5)^9$

*if you type this into your calc. you get a decimal! write it out instead to see the radicals "cancel"

$a_{10} = -2\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}\cdot\sqrt{5}$

$a_{10} = -2\cdot 5\cdot 5\cdot 5\cdot 5\cdot\sqrt{5}$

$a_{10} = -1250\sqrt{5}$